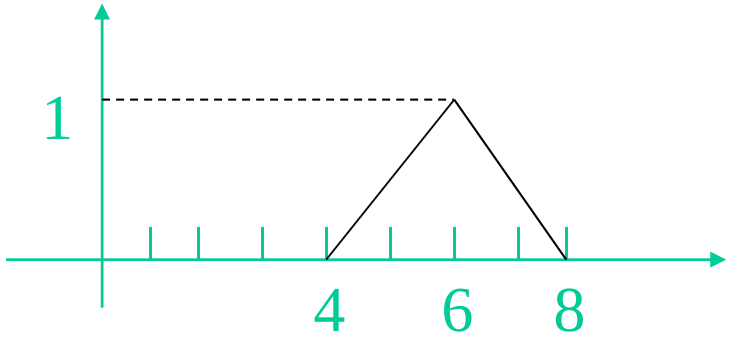
Fuzzy Arithmetic

To represent an inaccurate number

- Example
  - About two o'clock
  - Around six-thirty
  - Approximately six



- A number word and a linguistic modifier
- Fuzzy number
  - A fuzzy set defined in the set of real number
  - Degree 1 of central value
  - Membership degree decrease from 1 to 0 on both side

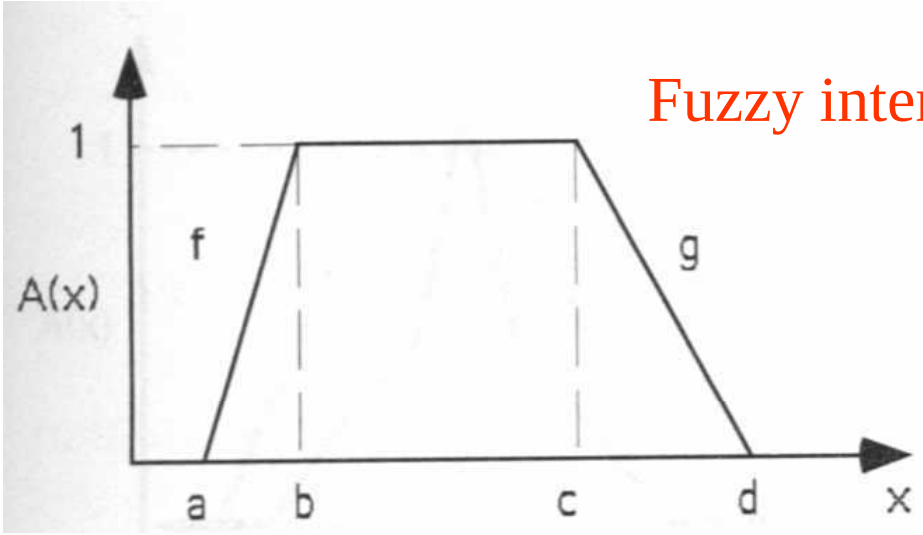
Fuzzy Numbers

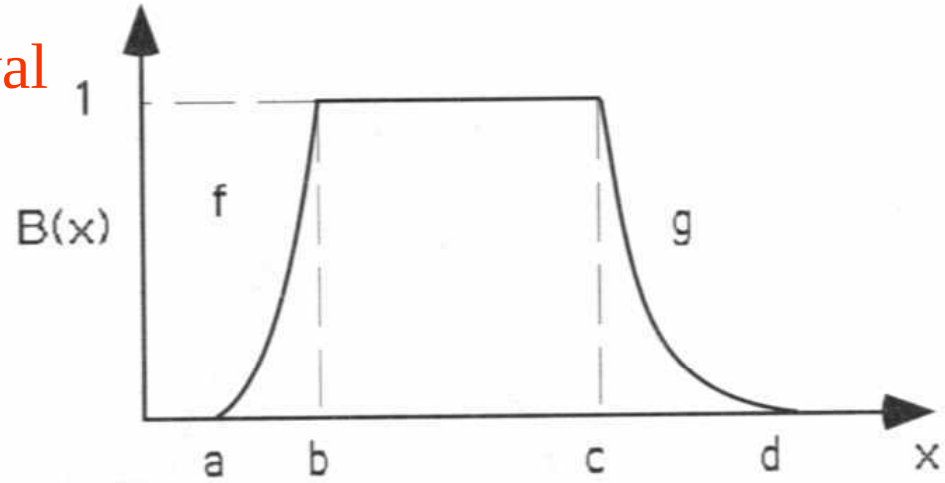
- The membership function of a fuzzy number is of the form A: $\Re \rightarrow [0,1]$

$$A(x) = \begin{cases} f(x) & \text{for } x \in [a,b] \\ 1 & \text{for } x \in [b,c] \\ g(x) & \text{for } x \in [c,d] \\ 0 & \text{for } x < a \text{ and } x > d \end{cases}$$

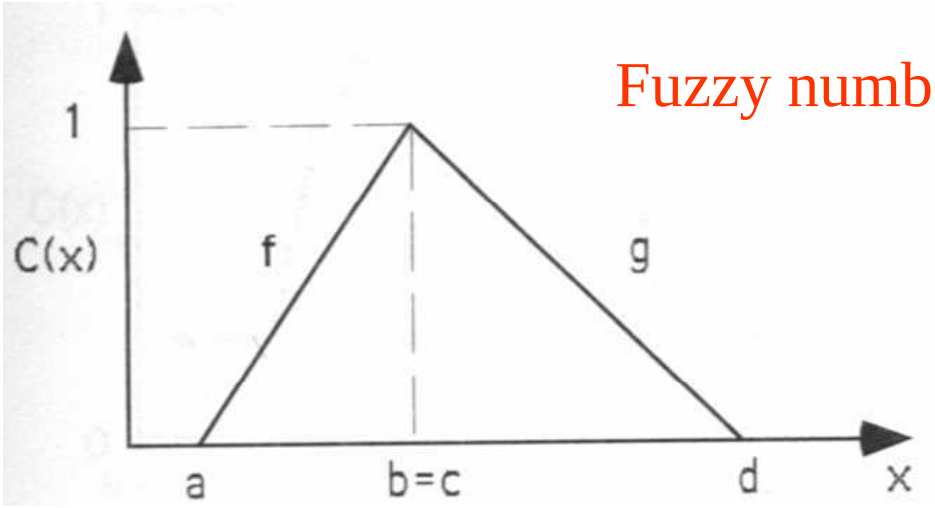
Examples of fuzzy numbers

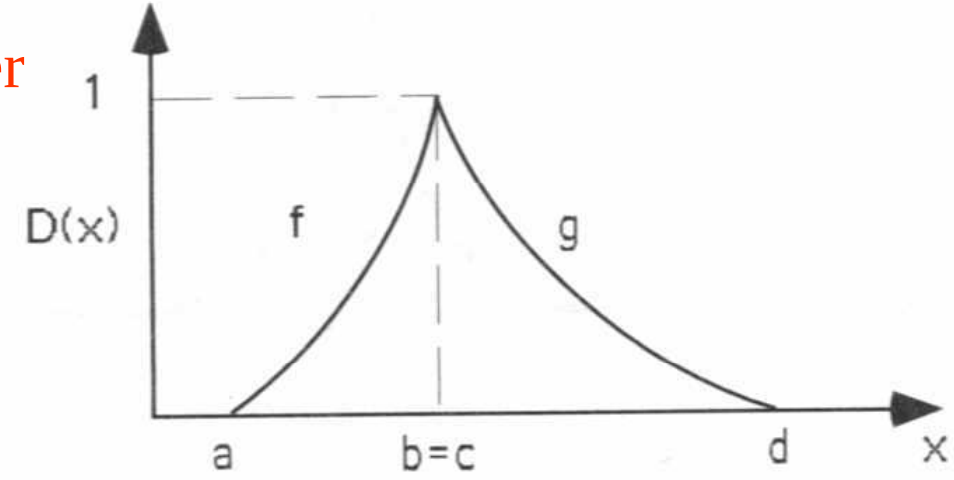
Fuzzy interval



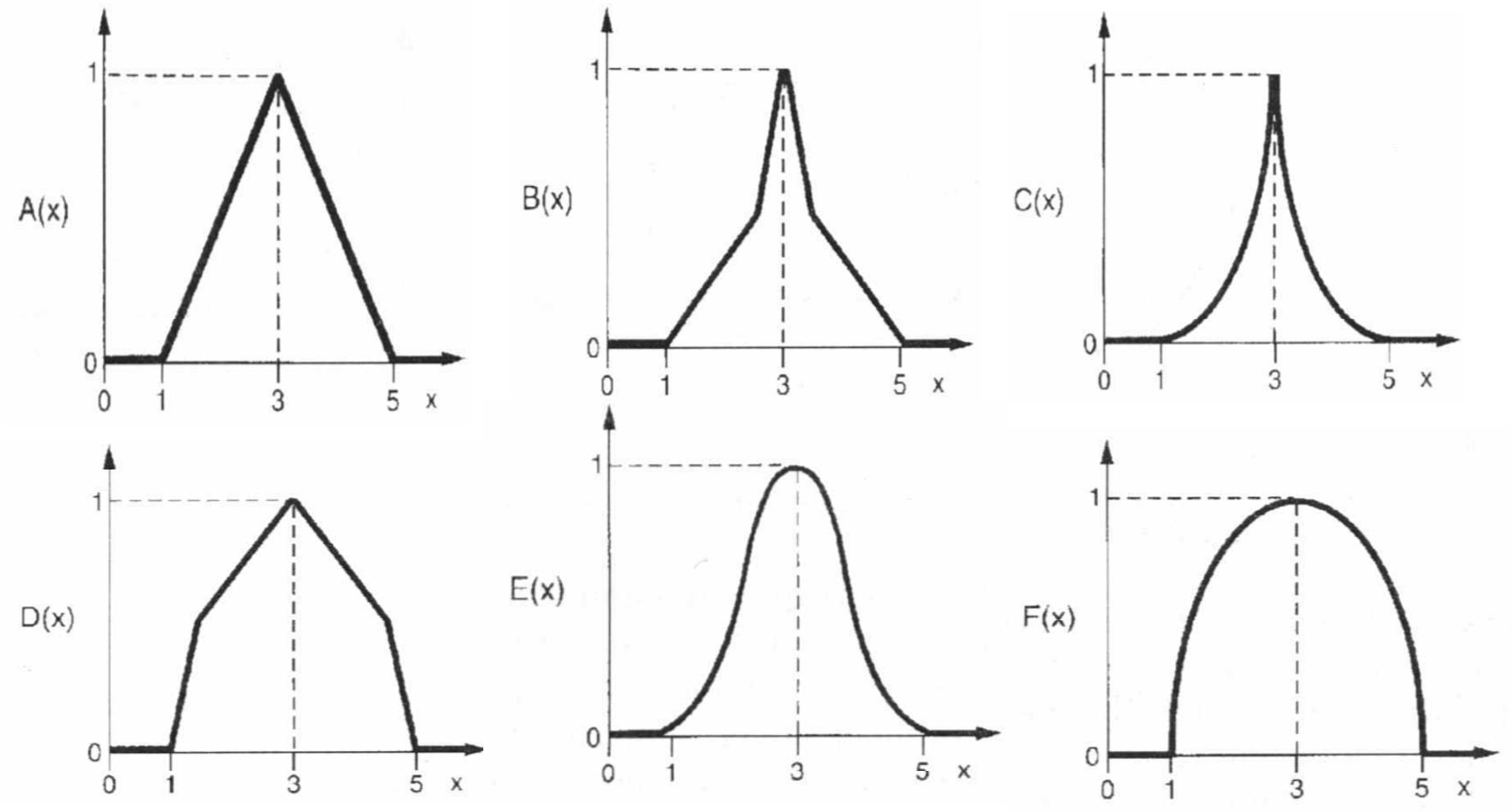


Fuzzy number

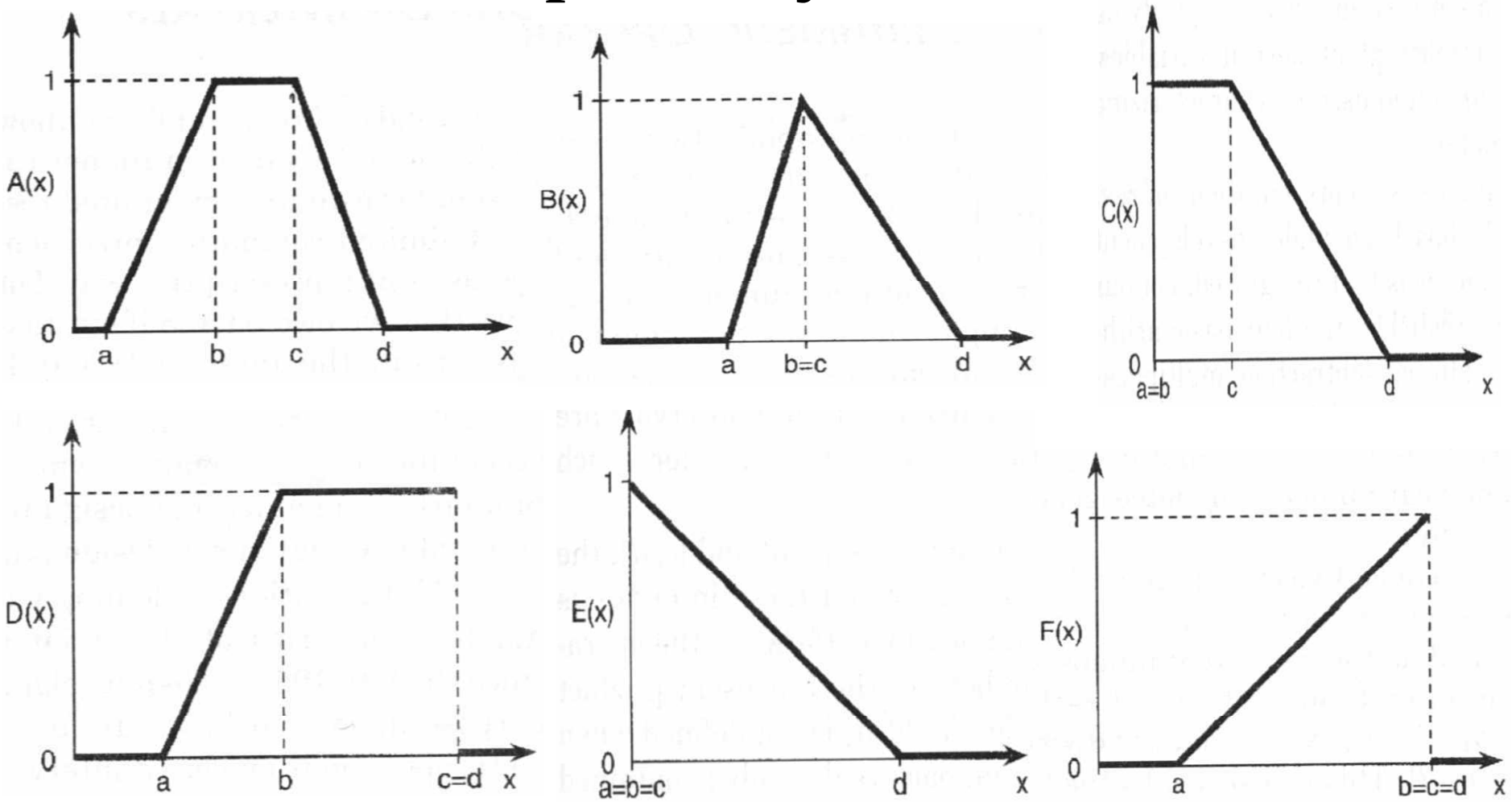




Fuzzy numbers of Around 3



Degenerated cases of trapezoidal-shape fuzzy number



Properties of fuzzy numbers

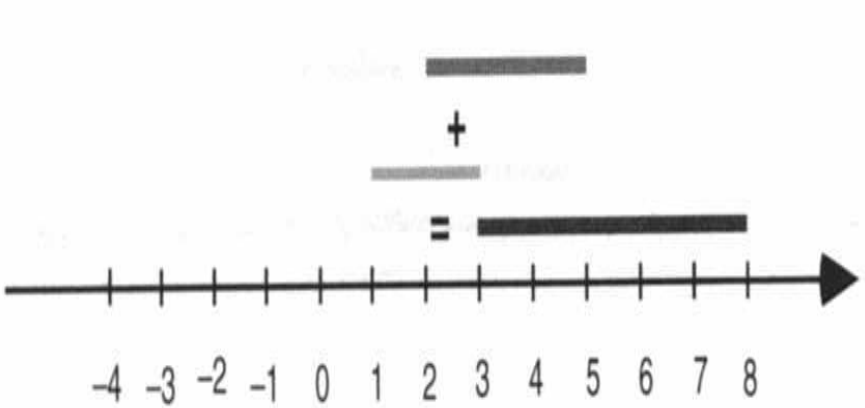
- Normal fuzzy sets
- The α-cuts of fuzzy number are closed intervals
- The support of every fuzzy number is the open interval (a,d)
- Convex fuzzy sets

Arithmetic operations on intervals

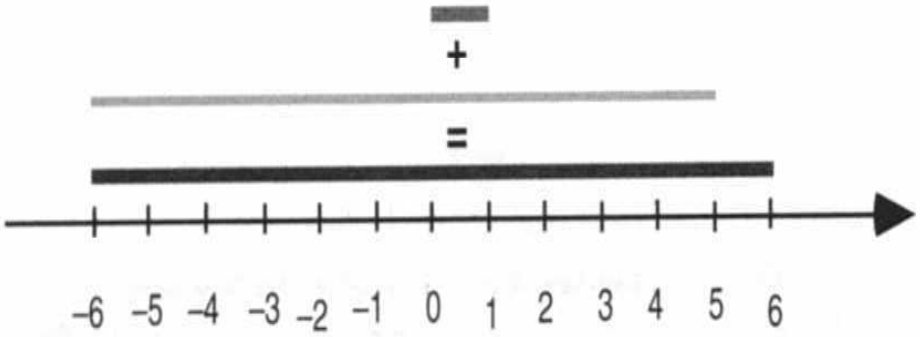
- Dealing with imprecise representation of real numbers in terms of closed intervals
- Arithmetic operation on closed interval [a,b] and [c,d]
  - Performing the operation on each ordered pair of real number in the Cartesian product $[a,b]\times[c,d]$
  - Addition(+), subtraction(-), multiplication(.), division(/)

Interval addition(+)

- [a, b] + [c, d] = [a+c, b+d]



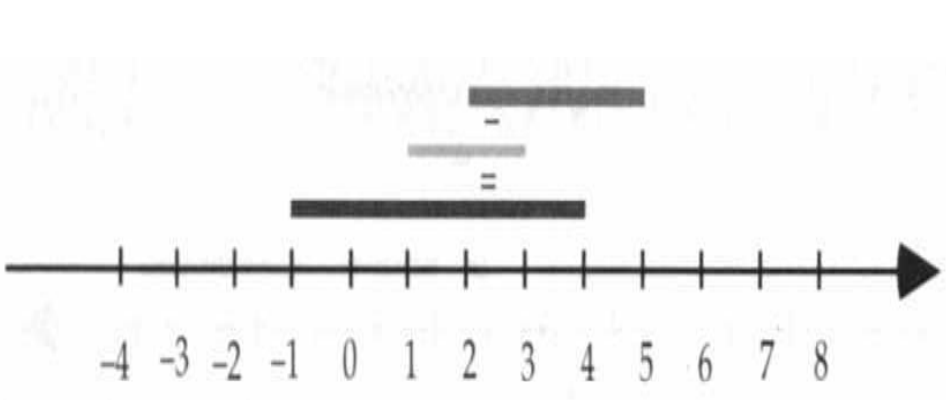
[2, 5] + [1, 3] = [3, 8]



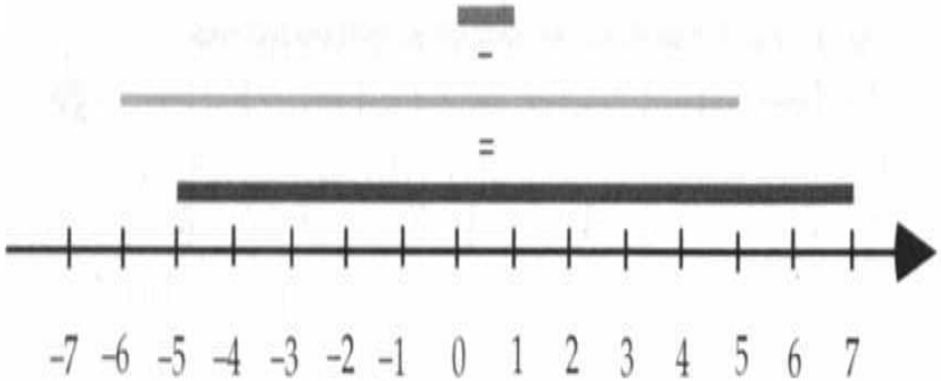
[0, 1] + [−6, 5] = [−6, 6]

Interval subtraction(-)

- [a, b] - [c, d] = [a-d, b-c]



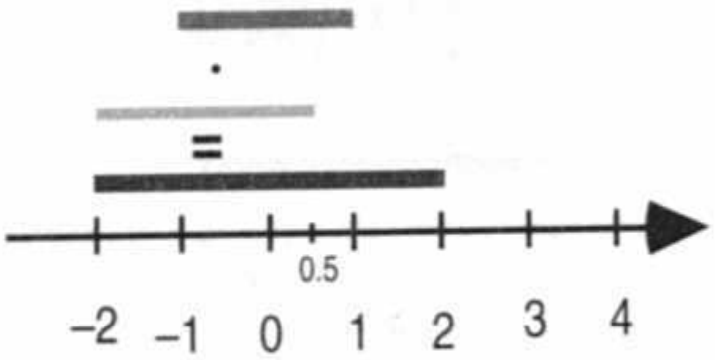
[2, 5] – [1, 3] = [–1, 4]



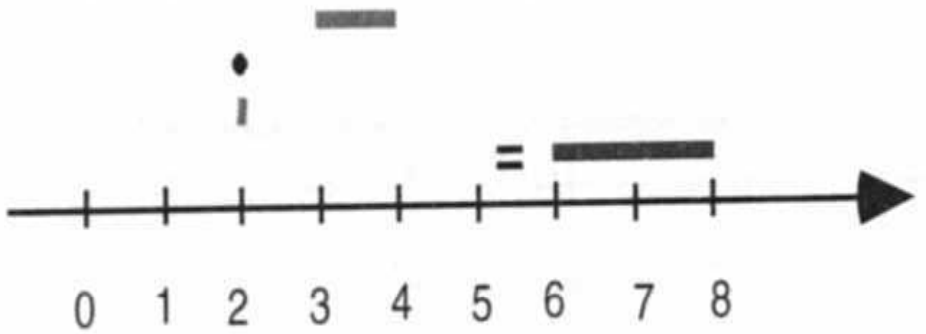
[0, 1] – [–6, 5] = [–5, 7]

Interval multiplication(.)

- [a, b] · [c, d] = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]



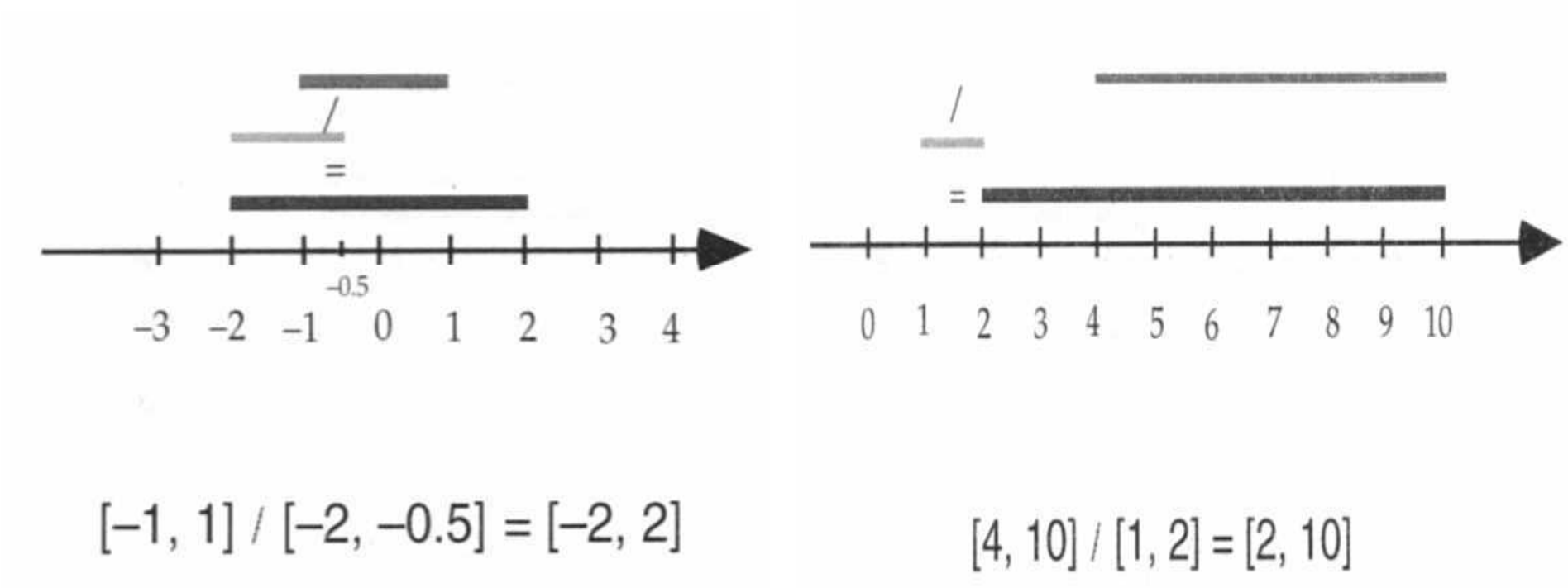
[−1, 1] · [−2, 0.5] = [−2, 2]



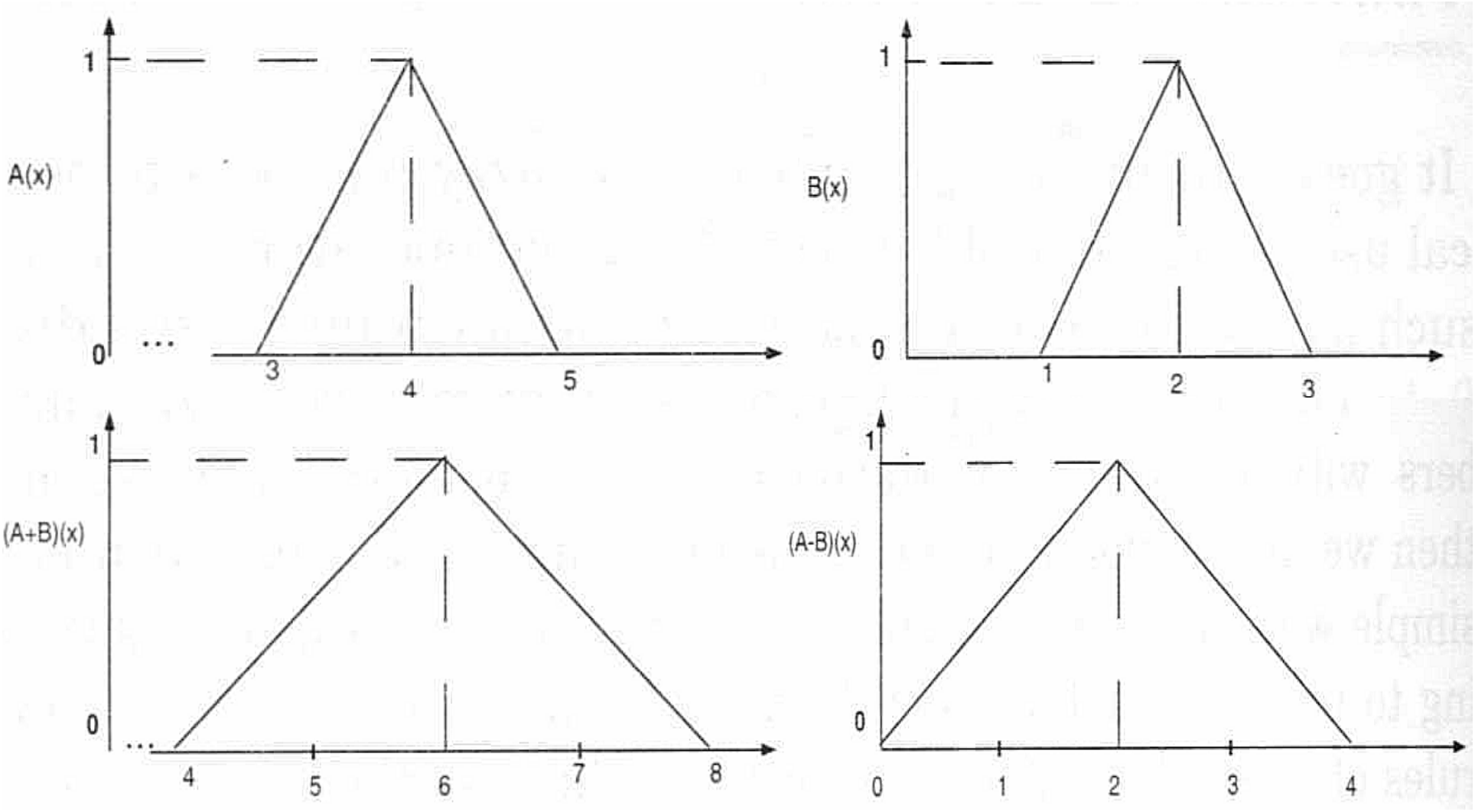
[3, 4] · [2, 2] = [6, 8]

Interval division(/)

- [a, b] / [c, d] = [a, b] · [1/d, 1/c]
  =[min(a/c, a/d, b/c, b/d), max(a/c, a/d, b/c, b/d)]



Simplified addition and subtraction of fuzzy number

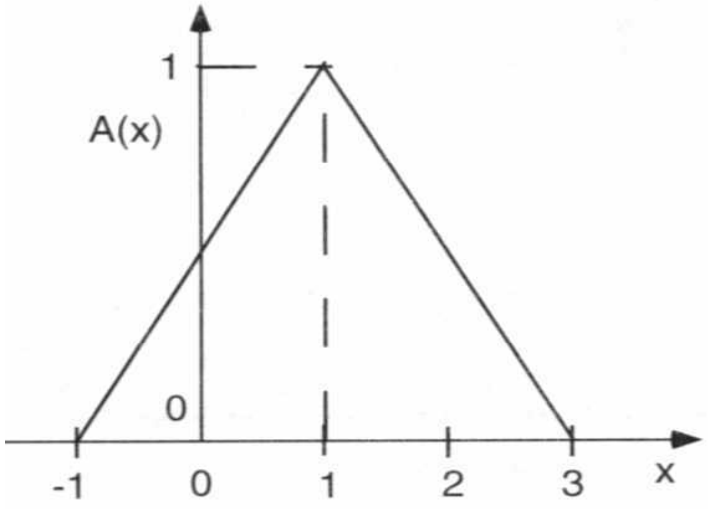


Arithmetic operations on fuzzy numbers

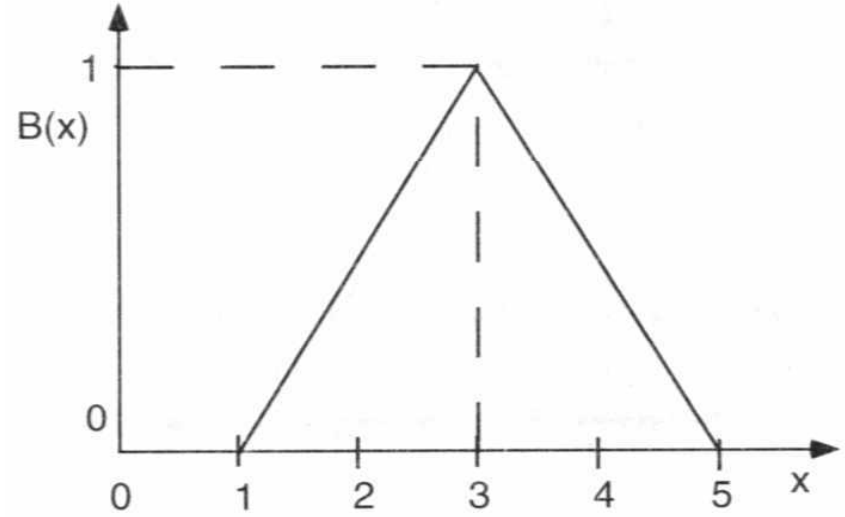
$^{\alpha}(A*B) = {}^{\alpha}A*{}^{\alpha}B$

- $A * B = \cup_{\alpha \in [0,1]} {}^{\alpha}(A*B) \cdot \alpha$
- Where * represents any of +, - , · and / operations

Fuzzy numbers for illustrating arithmetic operations



$$A(x) = \begin{cases} 0 & \text{for } x < -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 \le x \le 1 \\ (3-x)/2 & \text{for } 1 \le x \le 3 \end{cases}$$

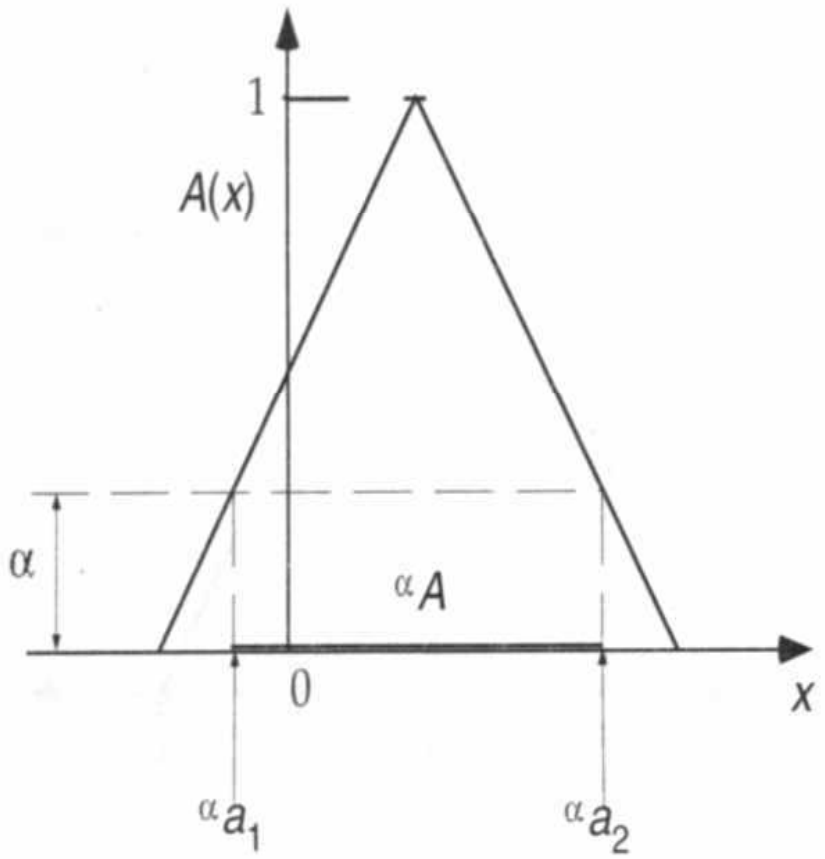


$$B(x) = \begin{cases} 0 & \text{for } x < 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 \le x \le 3 \\ (5-x)/2 & \text{for } 3 \le x \le 5 \end{cases}$$

Construction of α-cuts of A(x)

- $A(^{\alpha}a_1)=(^{\alpha}a_1+1)/2=\alpha$
- $A(^{\alpha}a_2)=(3-{}^{\alpha}a_2)/2=\alpha$

- $^{\alpha}A = [^{\alpha}a_1, {}^{\alpha}a_2]$
  $= [2\alpha-1, 3-2\alpha]$

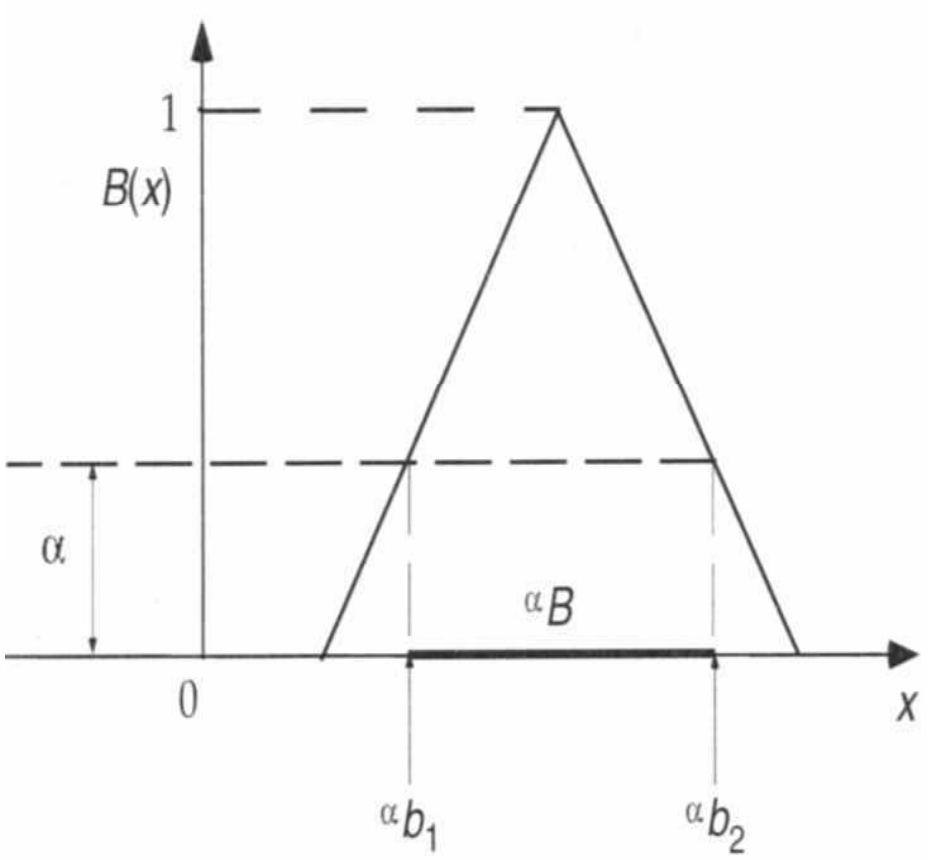


$$A(x) = \begin{cases} 0 & \text{for } x < -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 \le x \le 1 \\ (3-x)/2 & \text{for } 1 \le x \le 3 \end{cases}$$

Construction of α-cuts of B(*x*)

- $B({}^{\alpha}b_1)=({}^{\alpha}b_1 - 1)/2 = \alpha$
- $B({}^{\alpha}b_2)=(5 - {}^{\alpha}b_2)/2 = \alpha$

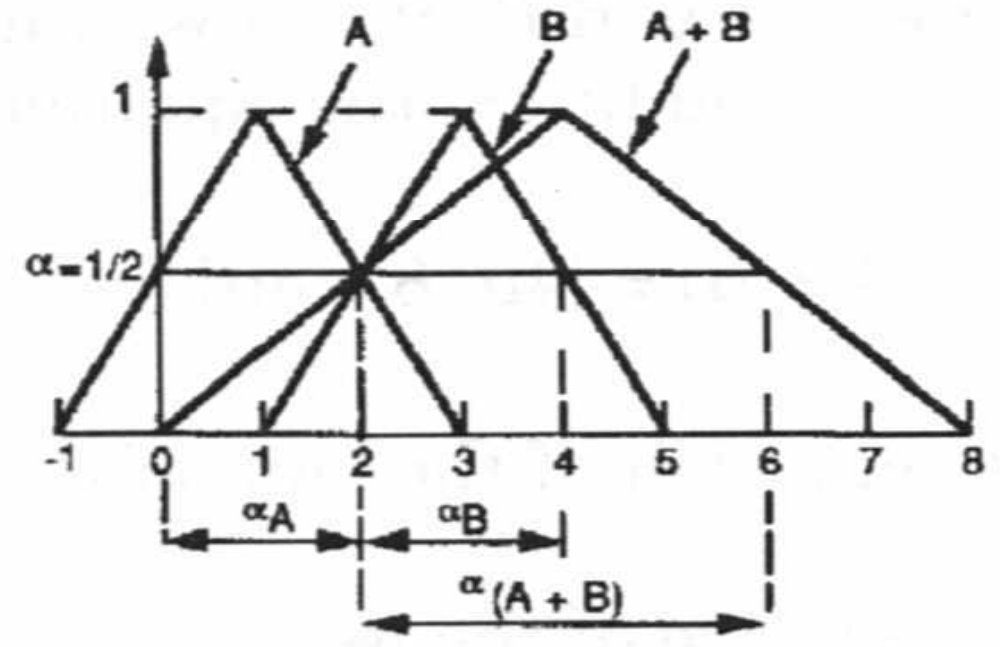
- ${}^{\alpha}B = [{}^{\alpha}b_1, {}^{\alpha}b_2]$
  $= [2\alpha+1, 5-2\alpha]$



$$B(x) = \begin{cases} 0 & \text{for } x < 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 \le x \le 3 \\ (5-x)/2 & \text{for } 3 \le x \le 5 \end{cases}$$

Fuzzy addition

- $^{\alpha}A = [2\alpha-1, 3-2\alpha]$
- $^{\alpha}B = [2\alpha+1, 5-2\alpha]$
- $^{\alpha}(A+B) = [4\alpha, 8-4\alpha]$

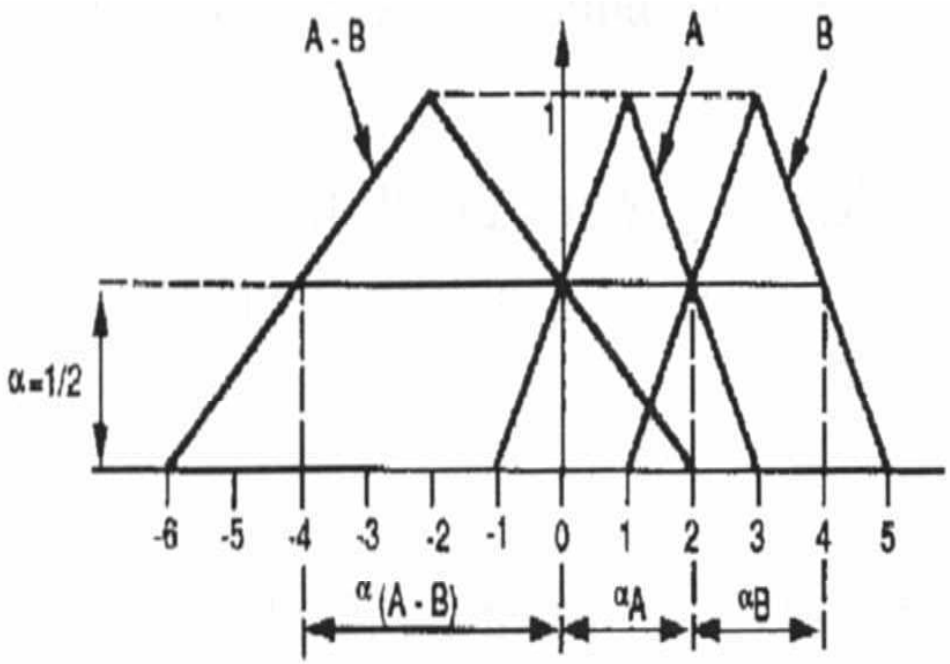


$$(A+B)(x) = \begin{cases} 0 & \text{for } x < 0 \text{ and } x > 8 \\ x/4 & \text{for } 0 \le x \le 4 \\ (8-x)/4 & \text{for } 4 \le x \le 8 \end{cases}$$

Fuzzy subtraction

- $^{\alpha}A = [2\alpha-1, 3-2\alpha]$
- $^{\alpha}B = [2\alpha+1, 5-2\alpha]$

- $^{\alpha}(A-B) = [4\alpha-6, 2-4\alpha]$



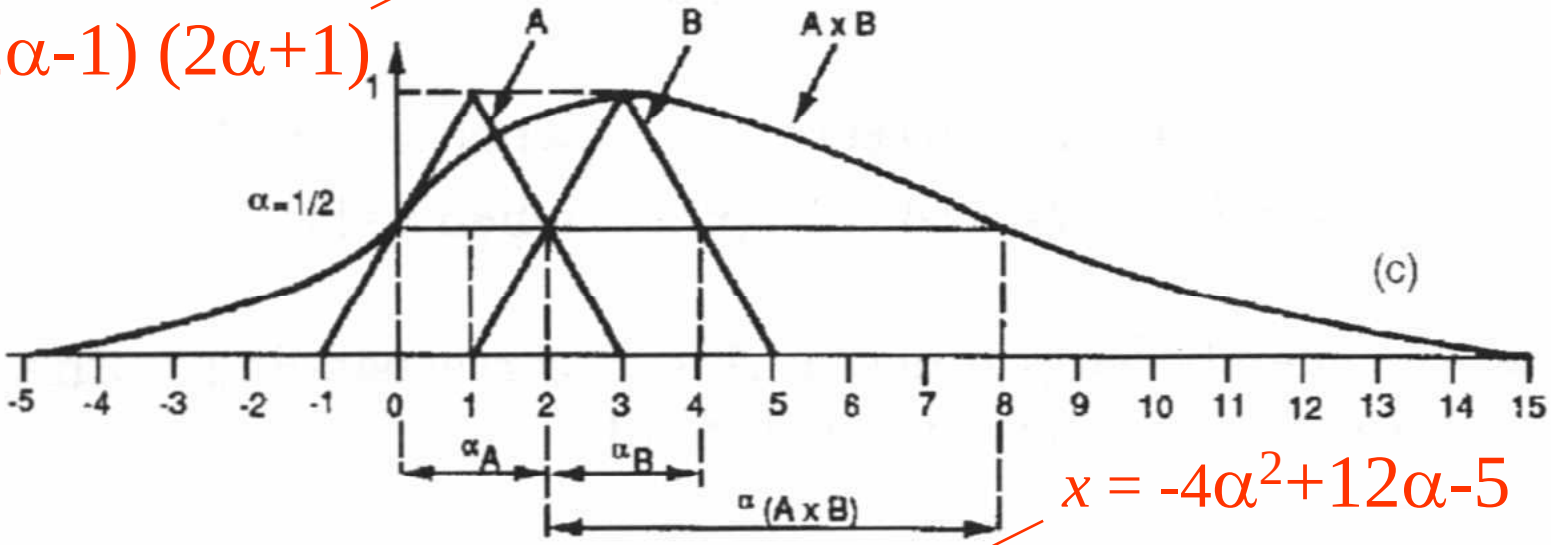
$$(A-B)(x) = \begin{cases} 0 & \text{for } x < -6 \text{ and } x > 2 \\ (x+6)/4 & \text{for } -6 \le x \le -2 \\ (2-x)/4 & \text{for } -2 \le x \le 2 \end{cases}$$

Fuzzy multiplication

$(2\alpha\text{-}1)(5\text{-}2\alpha)$ $(3\text{-}2\alpha)(5\text{-}2\alpha)$

$$^{a}(A \cdot B) = \begin{cases} [-4\alpha^2 + 12\alpha - 5, 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (0, 0.5] \\ [4\alpha^2 - 1, 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (0.5, 1] \end{cases}$$

$(2\alpha\text{-}1)$ $(2\alpha\text{+}1)$



$x = -4\alpha^2+12\alpha-5$

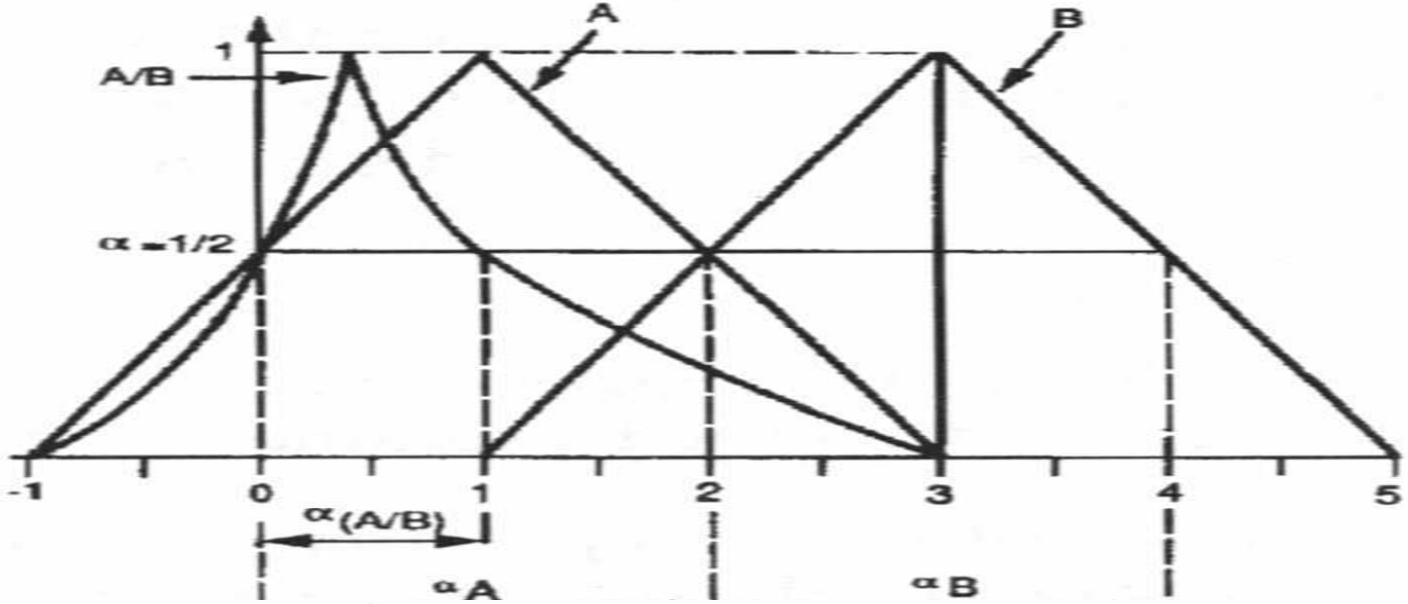
$$(A \ B)(x) = \begin{cases} 0 & \text{for } x < -5 \text{ and } x > 15 \\ [3 - (4 - x)^{1/2}]/2 & \text{for } -5 \le x < 0 \\ (1 + x)^{1/2}/2 & \text{for } 0 \le x < 3 \\ [4 - (1 + x)^{1/2}]/2 & \text{for } 3 \le x \le 15 \end{cases}$$

$x = 4\alpha^2\text{-}1$

$x = 4\alpha^2\text{-}16\alpha\text{+}15$

Fuzzy division

$$
{}^{\alpha}(A/B) = \begin{cases} [(2\alpha - 1)/(2\alpha + 1), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (0, 0.5] \\ [(2\alpha - 1)/(5 - 2\alpha), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (0.5, 1] \end{cases}
$$



$$
(A/B)(x) = \begin{cases} 0 & \text{for } x < -1 \text{ and } x > 3 \\ (x + 1)/(2 - 2x) & \text{for } -1 \le x < 0 \\ (5x + 1)/(2x + 2) & \text{for } 0 \le x < 1/3 \\ (3 - x)/(2x + 2) & \text{for } 1/3 \le x \le 3 \end{cases}
$$