Classical relations

A classical relation represents the presence or absence of association (birliktelik), interaction (etkileşim), or connection(bağlantı) between elements of two or more sets taken in a certain order.

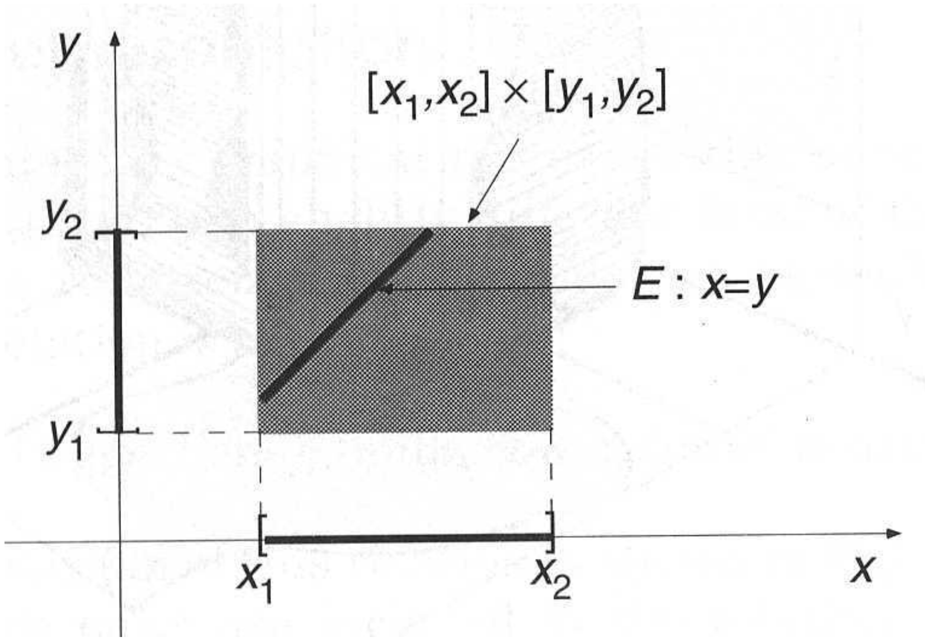
Cartesian product

- $A \times B = \{\langle a, b\rangle \mid a \in A \text{ and } b \in B\}$

- Example: $A=\{1,2\}$, $B=\{a,b,c\}$
- $A \times B = \{\langle 1,a\rangle, \langle 1,b\rangle, \langle 1,c\rangle, \langle 2,a\rangle, \langle 2,b\rangle, \langle 2,c\rangle\}$
- $B \times A = \{\langle a,1\rangle, \langle a,2\rangle, \langle b,1\rangle, \langle b,2\rangle, \langle c,1\rangle, \langle c,2\rangle\}$
- $A \times A = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}$
- $B \times B = \{\langle a,a\rangle, \langle a,b\rangle, \langle a,c\rangle, \langle b,a\rangle, \langle b,b\rangle, \langle b,c\rangle, \langle c,a\rangle, \langle c,b\rangle, \langle c,c\rangle\}$

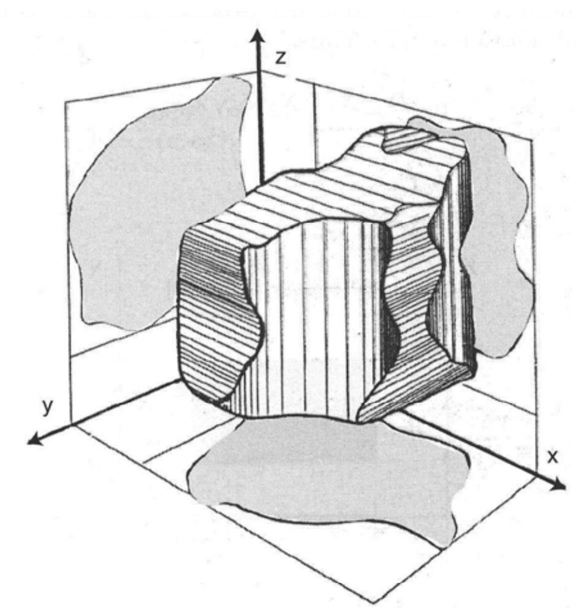
Equality relation between variables x and y

- E: $x=y$, where $x \in [x_1, x_2]$ and $y \in [y_1, y_2]$
- $E \subset [x_1, x_2] \times [y_1, y_2]$



n-dimensional relation

- a relation defined on the Cartesian product of n sets, $R \subseteq X_1 \times X_2 \times \ldots X_n$
  - Ternary: 3 sets
  - Quaternary: 4 sets
  - Quinary: 5 sets



Example

- Suppose that three sets A=B=C={1,2,3,4,5} find the ternary relation R on $A \times B \times C$ such that $a+b+c=5$, $a \in A$, $b \in B$, $c \in C$.
- R=?

Example

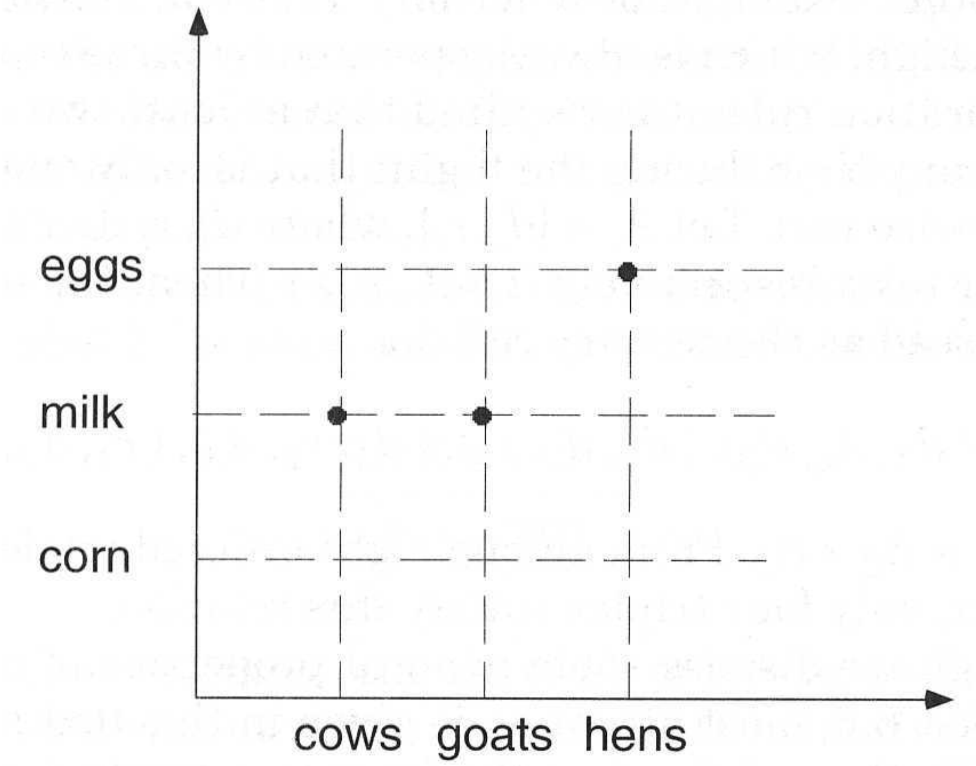
- Suppose that three sets $A=B=C=\{1,2,3,4,5\}$ find the ternary relation R on $A \times B \times C$ such that $a+b+c=5$, $a\in A$, $b\in B$, $c\in C$.
- $R=\{<1,1,3>,<1,3,1>,<3,1,1>,<1,2,2>,<2,1,2>,<2,2,1>\}$

Representations

- The representations of binary relations
  - List
  - Coordinate
  - Matrix
  - Mappings
  - Directed graph

Coordinate representation

- R ={<eggs, hens>, <milk,cows>, <milks,goats>}



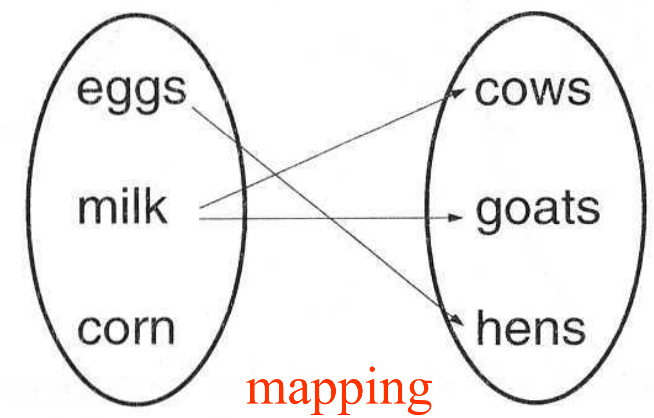
Matrix representation

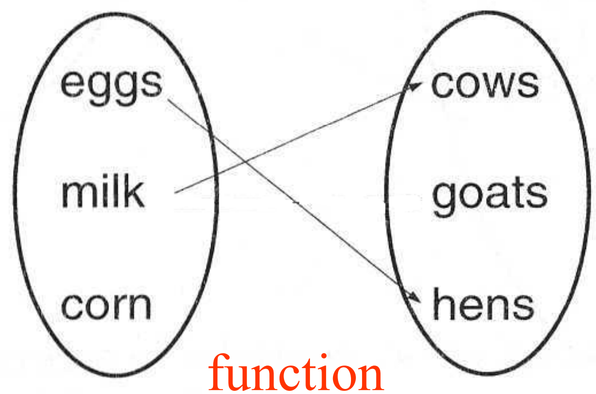
- 0 if the relation does not exist between the two individuals and 1 if it does.

R	cows	goats	hens
eggs	0	0	1
milk	1	1	0
corn	0	0	0

Mappings

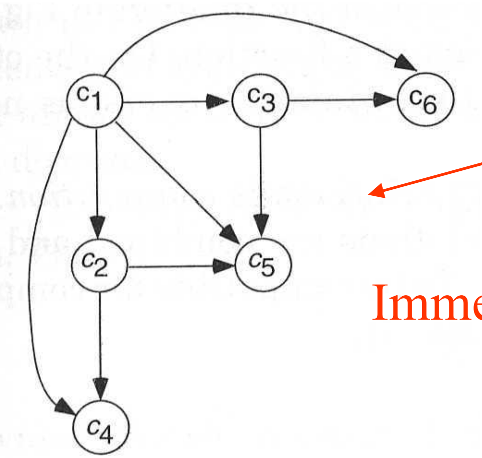
- Functions: binary relations in which no element of the first set is mapped to more than one element of the second set.





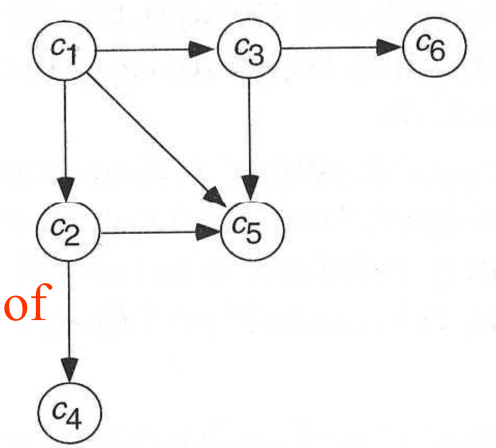
Directed graph

- Properties of directed graph
  - Each element of the set X is represented by a node in the diagram
  - Directed connections between nodes indicate pairs of elements that are included in the relation



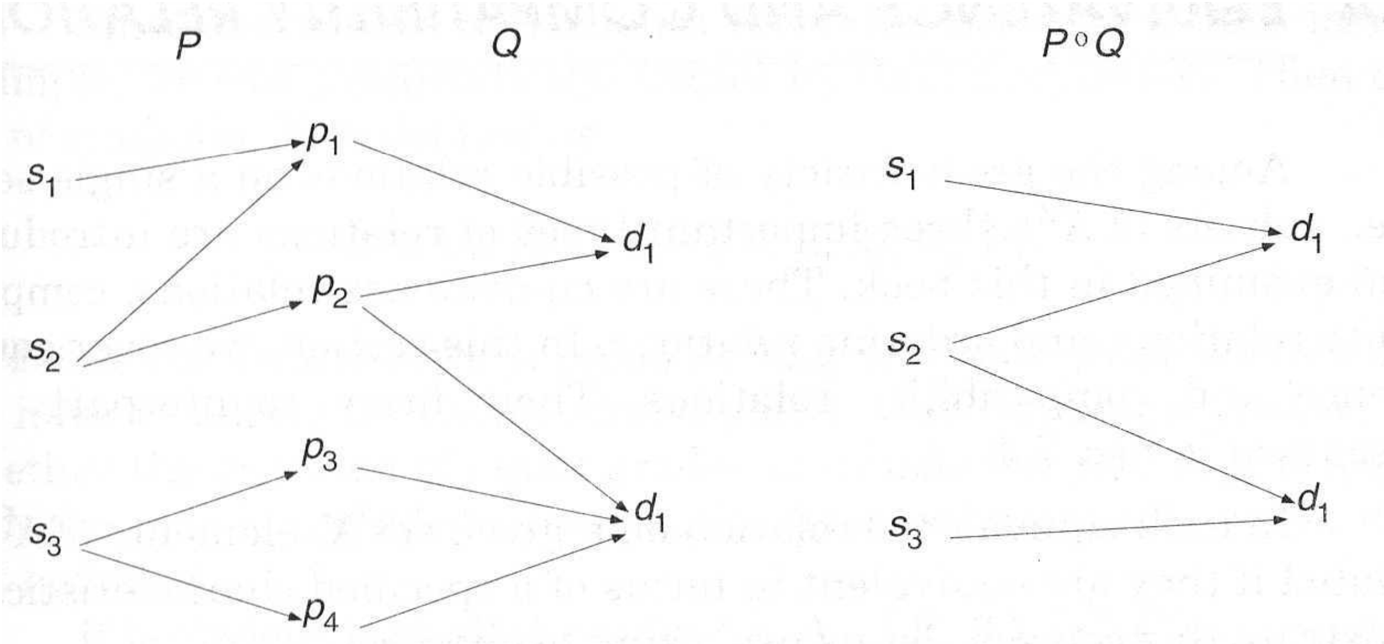
Prerequisite of

Immediate prerequisite of



Operations on binary relations

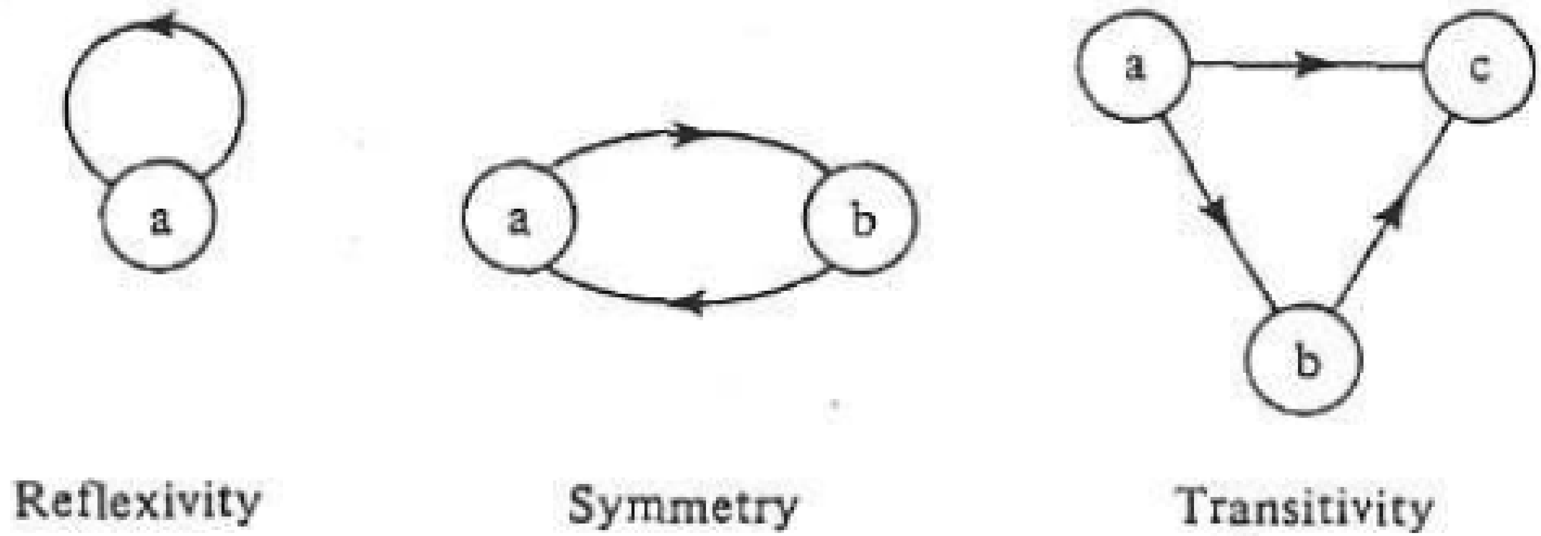
- Given binary relation $R \subseteq X \times Y$
  - Inverse $R^{-1} \subseteq Y \times X$
- Given binary relations $P \subseteq X \times Y$, $Q \subseteq Y \times Z$
  - Composition on P and Q = $P \circ Q = R \subseteq X \times Z$



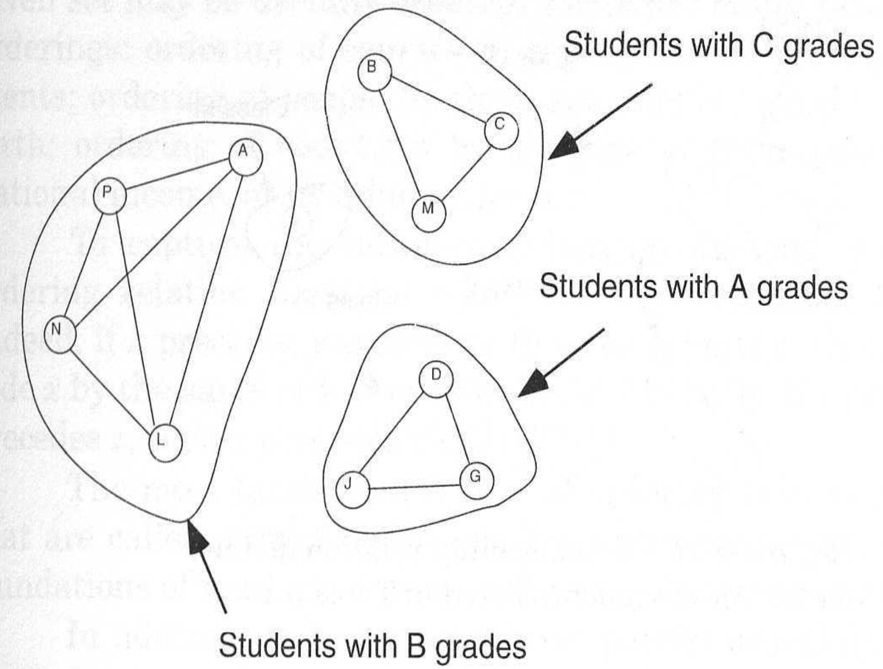
Equivalence and compatibility relation

- Equivalence (Denklik): Any binary relation that satisfies reflexive, symmetric and transitive properties.
  - R is Reflexive $\Leftrightarrow$ $<x, x> \in R$ for each $x \in R$
  - R is symmetric $\Leftrightarrow$ both $<x, y> \in R$ and $<y, x> \in R$ or both not.
  - R is transitive $\Leftrightarrow$ for any three elements x, y, z in X, $<x, z> \in R$ whenever $<x, y> \in R$ and $<y, z> \in R$
- Compatibility (Uyumluluk): satisfy reflexive and symmetric

Equivalence



Student	Grade	Major	Age	Full-time/ part-time
Alan	B	Biology	19	Full-time
Bob	C	Physics	19	Full-time
Cliff	C	Mathematics	20	Part-time
Debby	A	Mathematics	19	Full-time
George	A	Mathematics	19	Full-time
Jane	A	Business	21	Part-time
Lisa	B	Chemistry	21	Part-time
Mary	C	Biology	19	Full-time
Nancy	B	Biology	19	Full-time
Paul	B	Business	21	Part-time



R	*A*	*B*	*C*	*D*	*G*	*J*	*L*	*M*	*N*	*P*
A	1	0	0	0	0	0	1	0	1	1
B	0	1	1	0	0	0	0	1	0	0
C	0	1	1	0	0	0	0	1	0	0
D	0	0	0	1	1	1	0	0	0	0
G	0	0	0	1	1	1	0	0	0	0
J	0	0	0	1	1	1	0	0	0	0
L	1	0	0	0	0	0	1	0	1	1
M	0	1	1	0	0	0	0	1	0	0
N	1	0	0	0	0	0	1	0	1	1
P	1	0	0	0	0	0	1	0	1	1

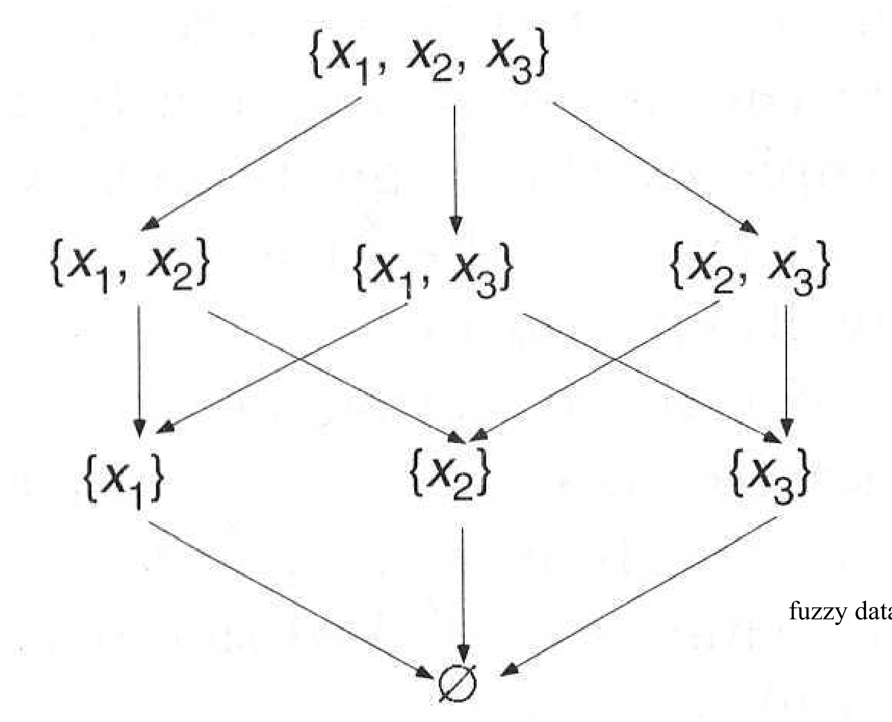
Partial ordering (Kısmi sıralama)

predecessor

- Relations that satisfy reflexive, transitive and anti-symmetric, denoted by $x \leq y$

successor

  - R on X is anti-symmetric $\Leftrightarrow$ for any x and y in X, if $\langle x,y \rangle \in R$ and $\langle y, x \rangle \in R$ then $x=y$.



$A \subseteq B \Leftrightarrow A \leq B$

Hasse diagram:

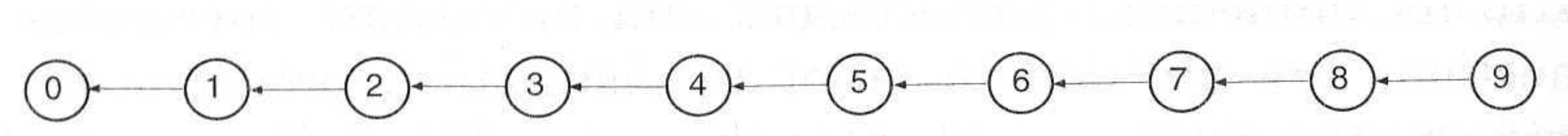
The power set of $\{ x, y, z \}$ partially ordered by inclusion

Linear ordering

- Partial ordering relation satisfy that either a≤b or b≤a for all a, b ∈X.

Greatest lower bound
l= inf X

Least upper bound
u= sup X



min X

max X

fuzzy database modeling

max, min, supremum, infimum

$\max\{2, 5, 7, 10\} = 10$

$\min\{2, 5, 7, 10\} = 2$

$\sup\{2, 5, 7, 10\} = 10$

$\inf\{2, 5, 7, 10\} = 2$

For a set with finite elements

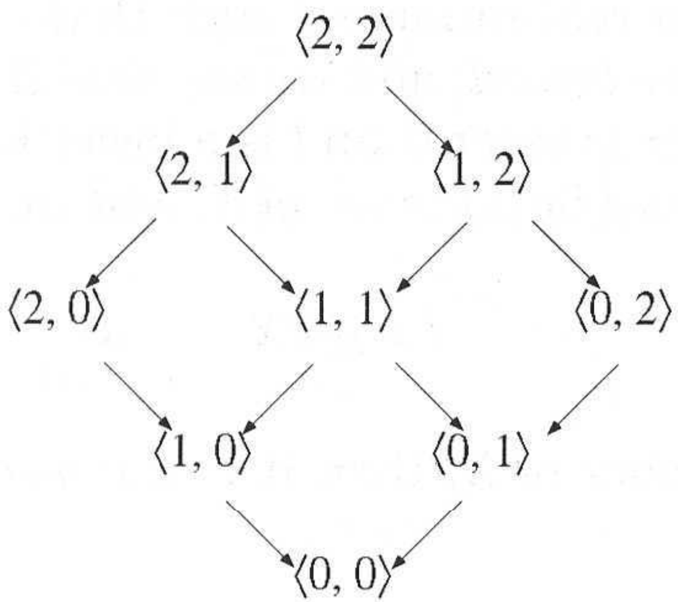
$\sup\{0.0 < x < 10.0\} = 10.0$

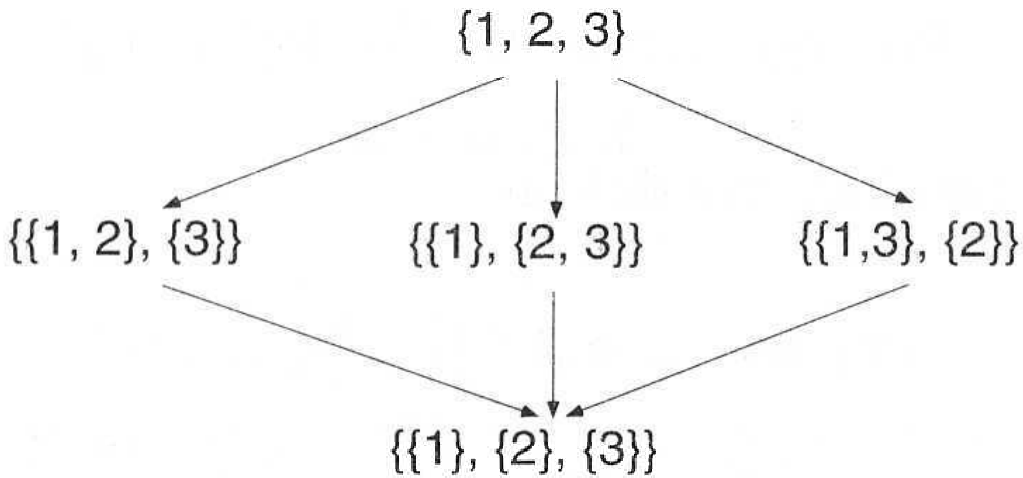
$\inf\{0.0 < x < 10.0\} = 0.0$

For an infinite set

fuzzy database modeling

Examples of partial ordering

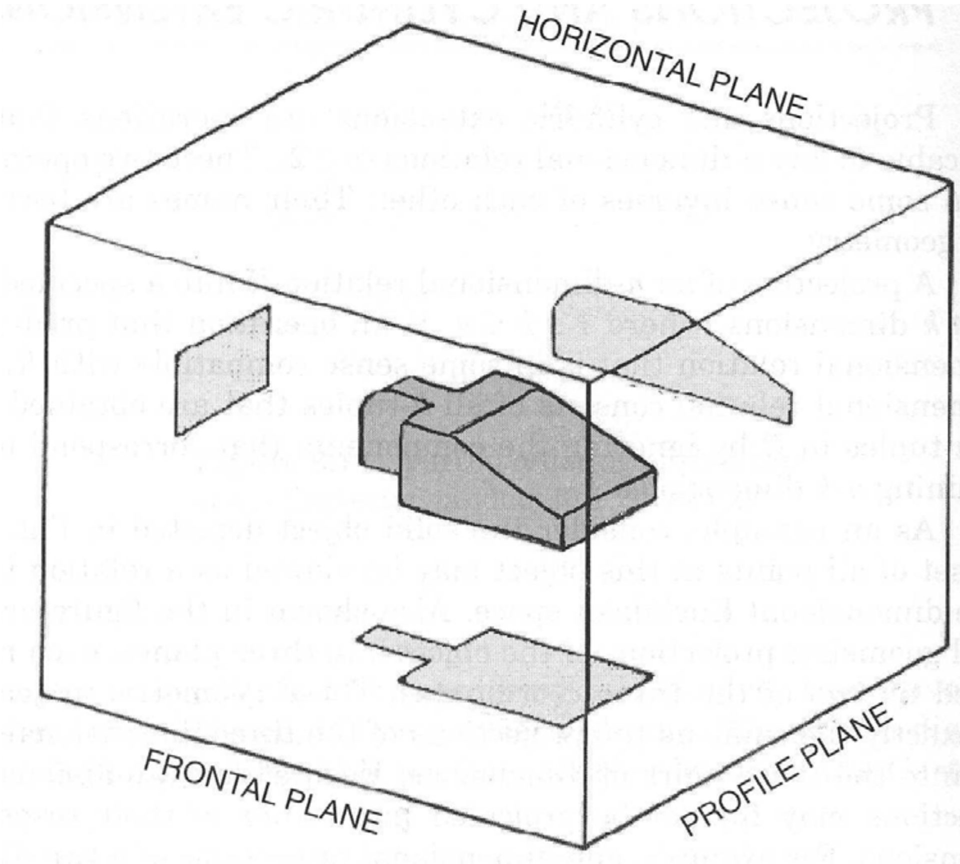




Partial ordering of elements of the Cartesian product $\{0,1,2\} \times \{0,1,2\}$

Refinement ordering of partition of the set $\{1,2,3\}$

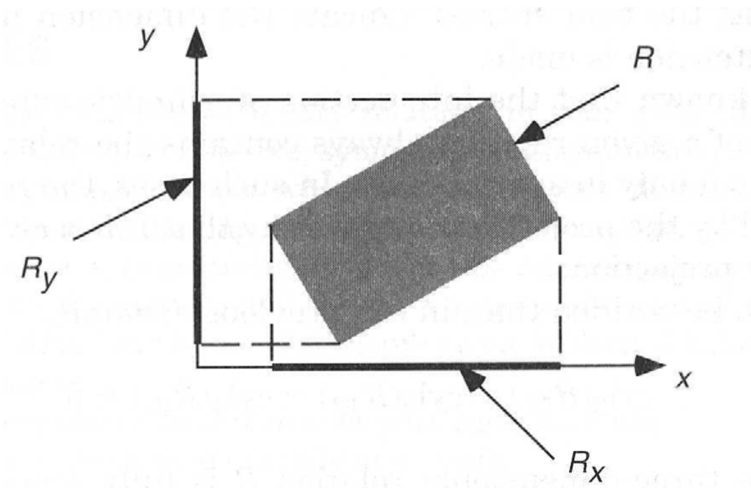
Projection (İzdüşüm)



n dimensions

↓

k dimensions



Example of projection

R:

0	0	1
0	1	1
1	1	0
1	1	1

R_{12}:

0	0
0	1
1	1

R_{13}:

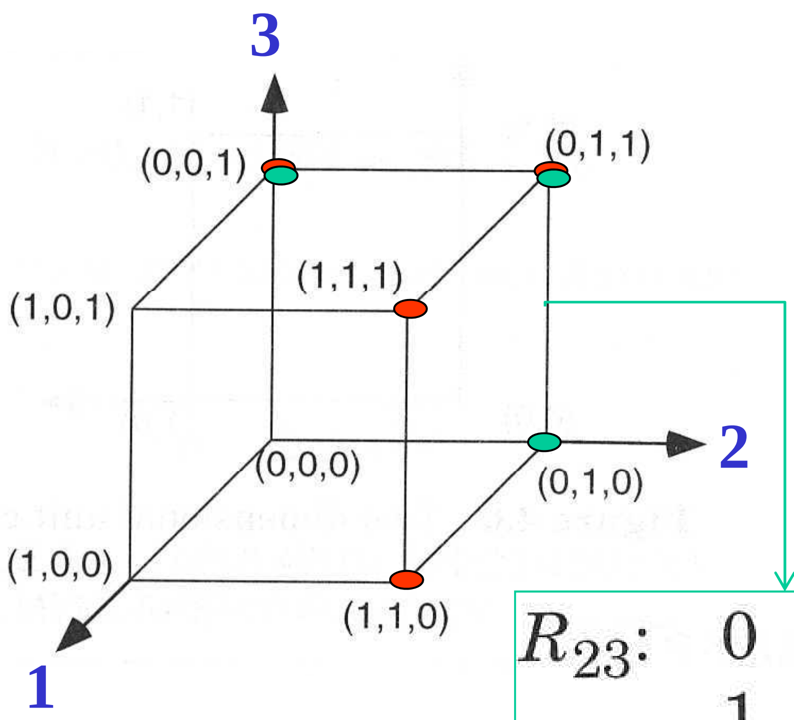
0	1
1	0
1	1

R_{23}:

0	1
1	1
1	0

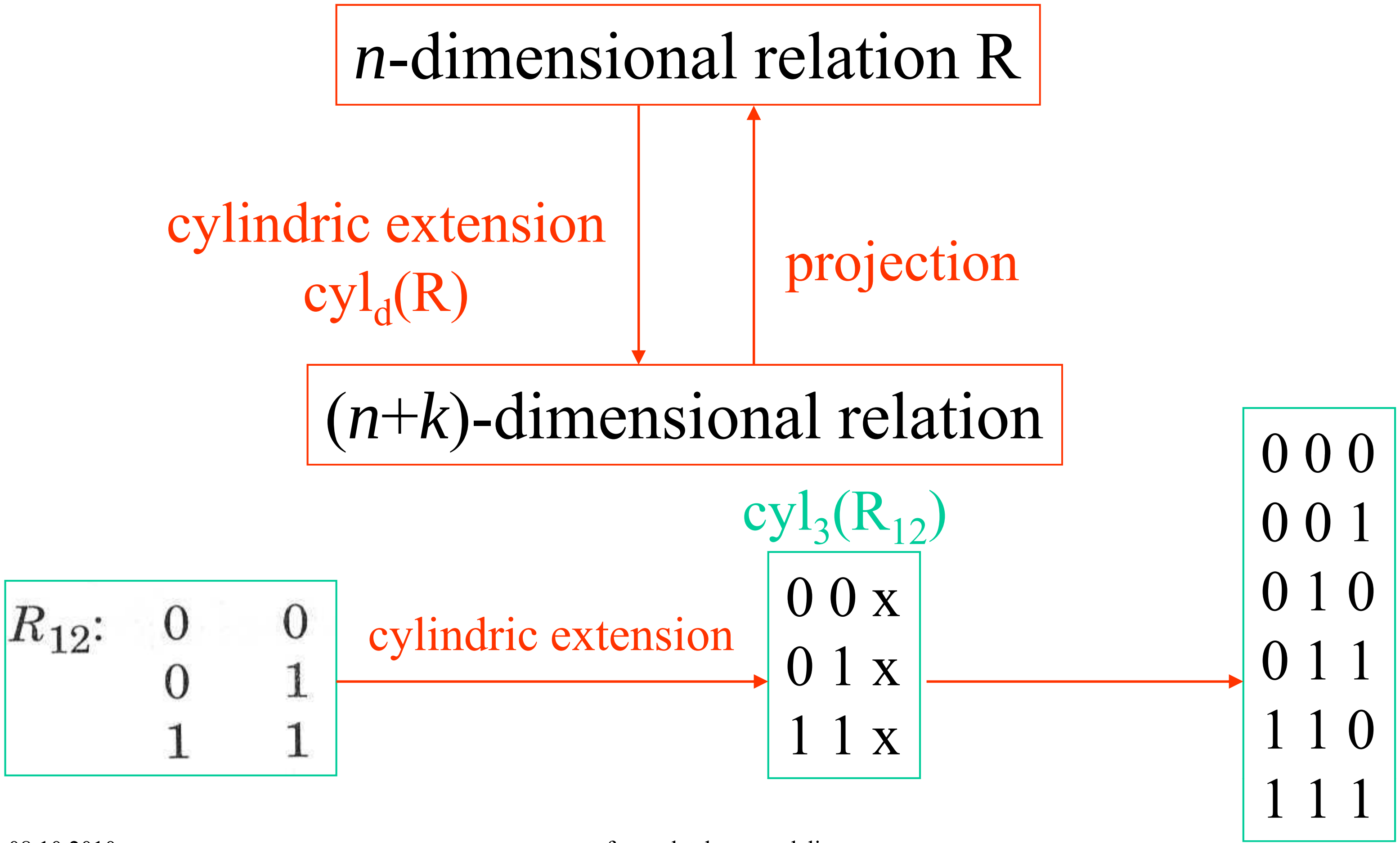
Example of projection

R:	0	0	1
	0	1	1
	1	1	0
	1	1	1



R_{23}:	0	1
	1	1
	1	0

Cylindric extension



Example of cylindric extension

- $cyl_3(R_{12}) \cap cyl_2(R_{13}) \cap cyl_1(R_{23}) = R$

0	0	1
0	1	1
1	1	0
1	1	1

$cyl_3(R_{12})$:

0	0	0
0	0	1
0	1	0
0	1	1
1	1	0
1	1	1

$cyl_2(R_{13})$:

0	0	1
0	1	1
1	0	0
1	1	0
1	0	1
1	1	1

$cyl_1(R_{23})$:

0	0	1
1	0	1
0	1	1
1	1	1
0	1	0
1	1	0

Projection and cylindric extension