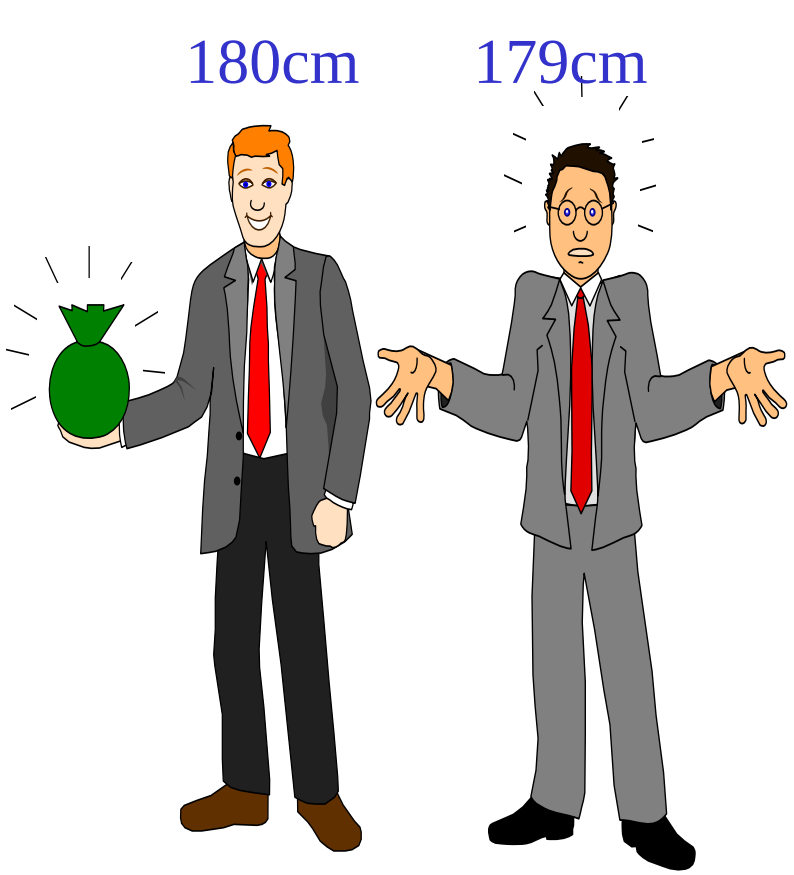
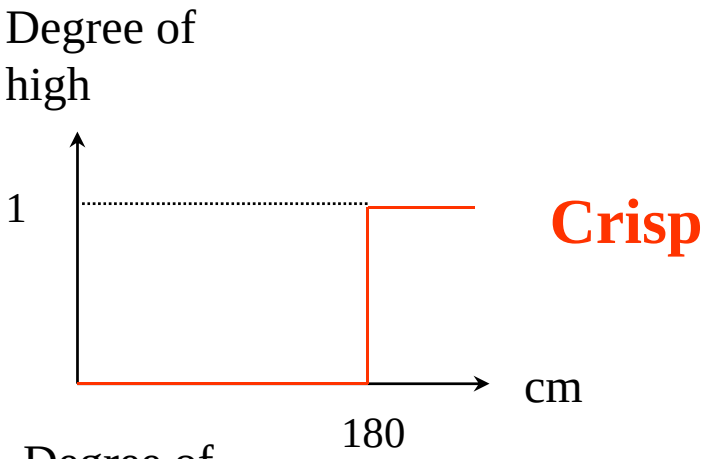
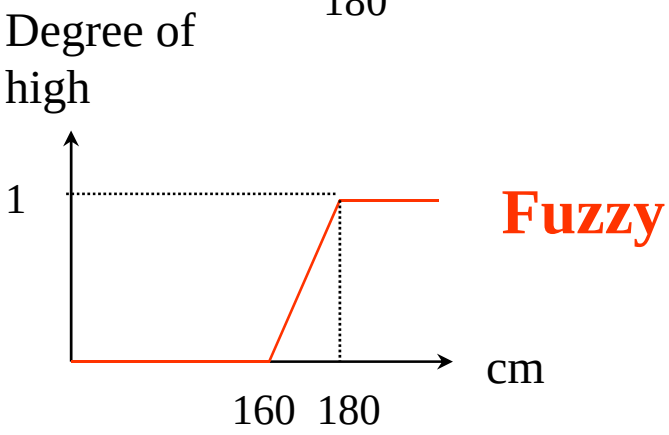
Fuzzy sets: Basic concepts and properties

Fuzzy and crisp





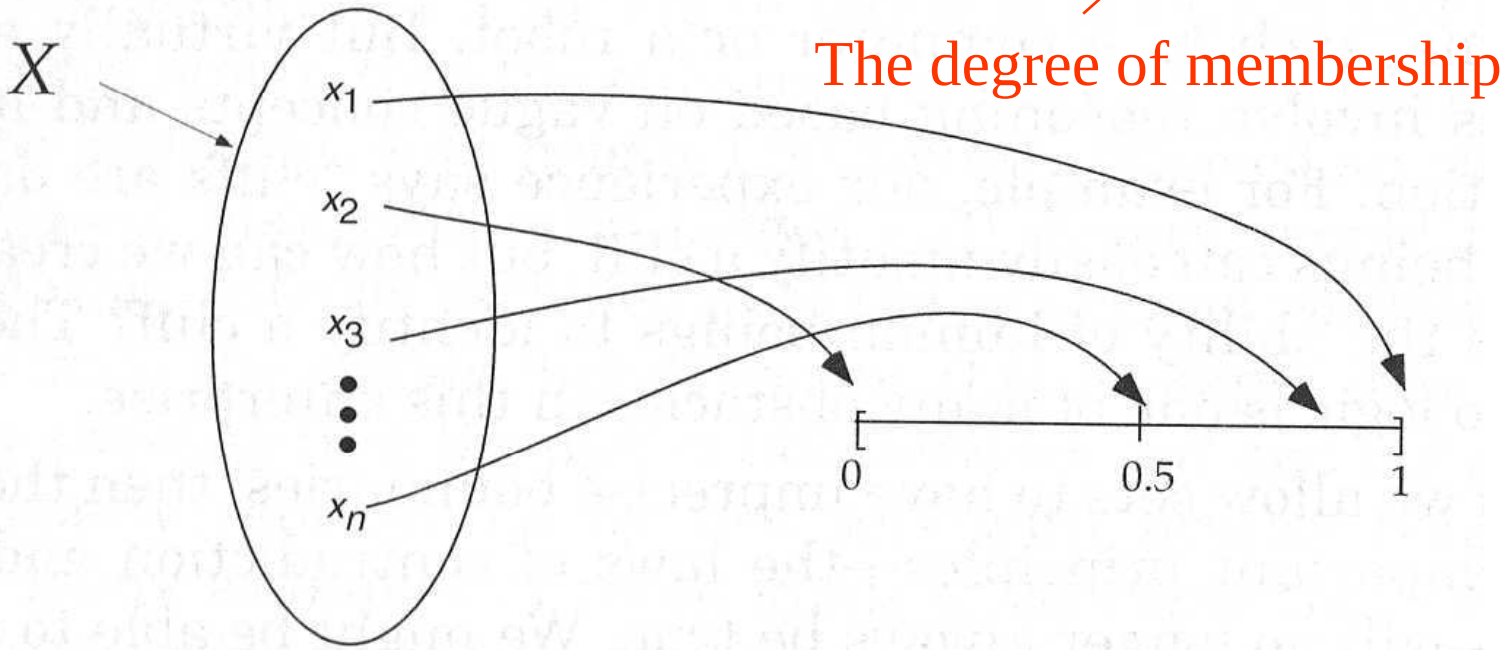


Membership functions

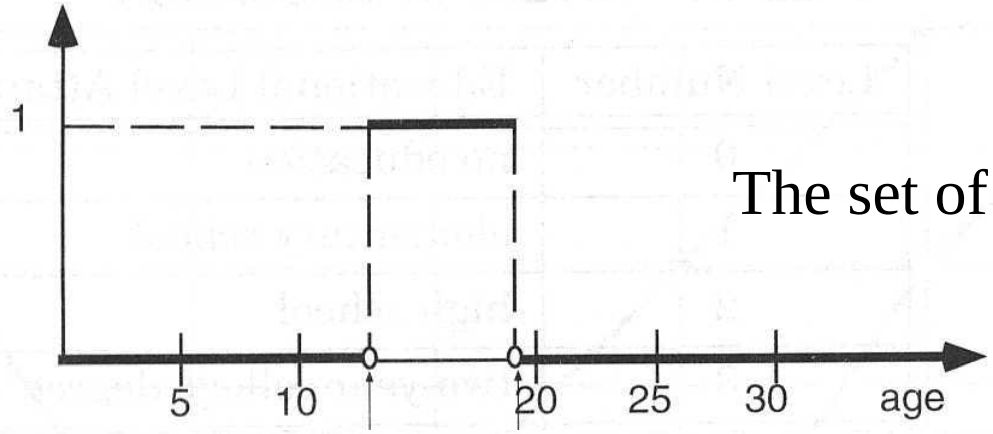
- Assign to each element x of X a number A(x)

$$A: X \rightarrow [0, 1]$$

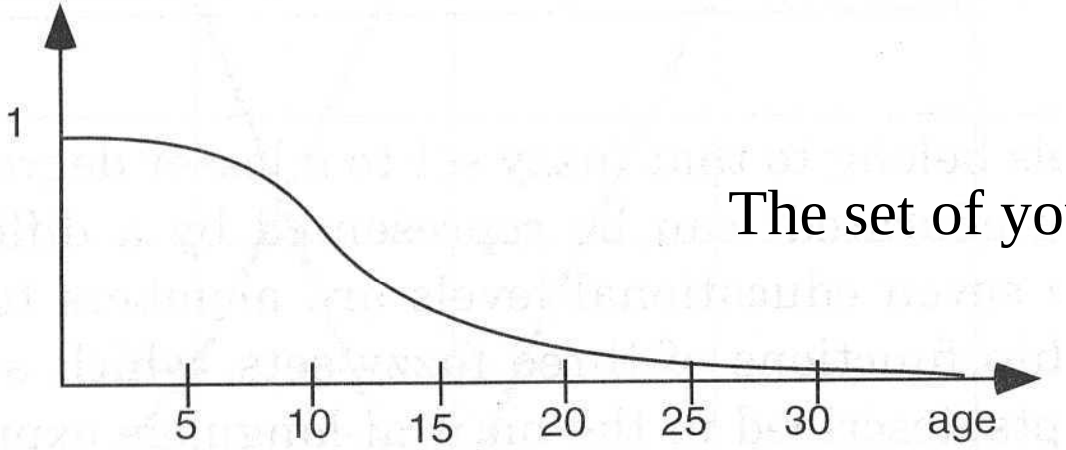
The degree of membership



examples



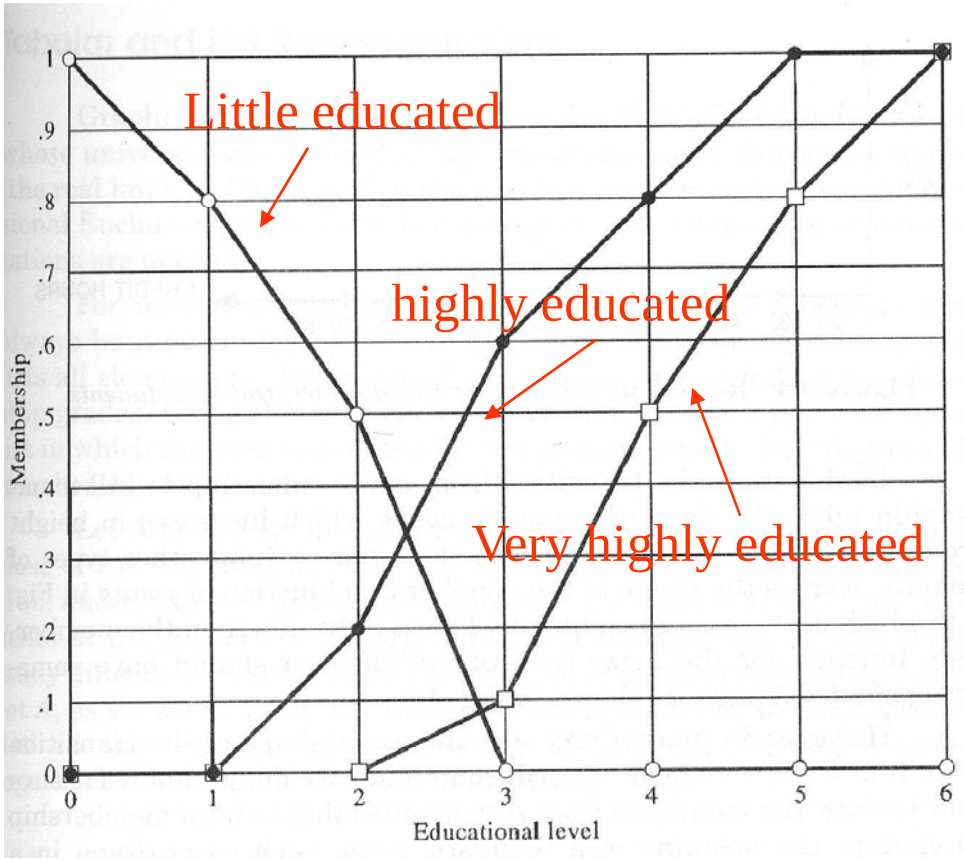
The set of teenagers



The set of young people

Graphical representation

Level Number	Educational Level Attained
0	no education
1	elementary school
2	high school
3	two-year college degree
4	bachelor's degree
5	master's degree
6	doctoral degree



Tabular and list representations

- List representation of very high educated

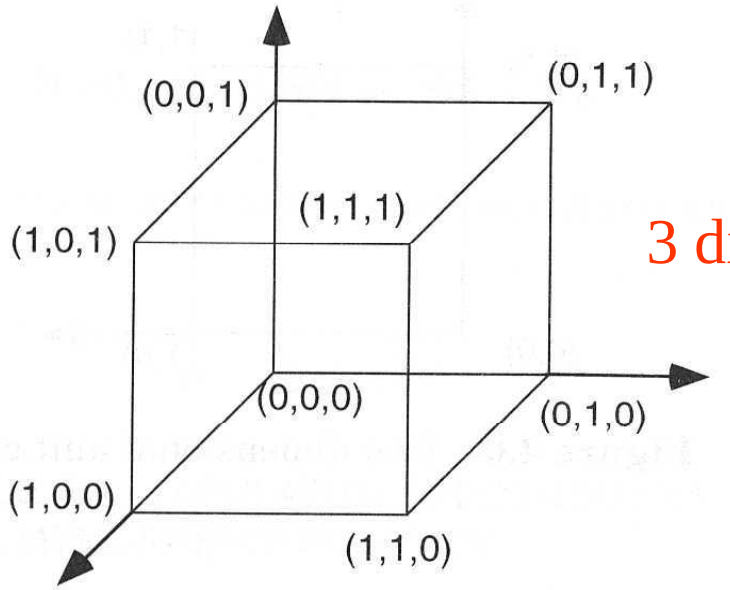
B = 0/0 + 0/1 + 0/2 + 0.1/3 + 0.5/4 +0.8/5 + 1/6

- General notation $A=\sum A(x)/x$
- Tabular (çizelge) representations

level	membership
0	0
1	0
2	0
3	0.1
4	0.5
5	0.8
6	1

Geometric representation

- All fuzzy sets that can be defined on a universal set with *n* elements can be represented by vertices on *n*-cube.



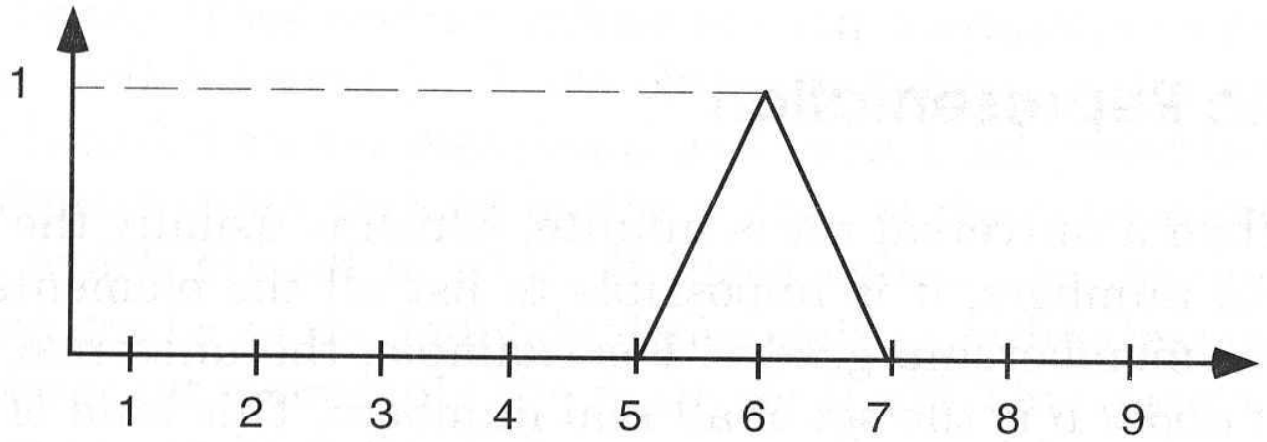
3 dimension unit cube

X={x_1, x_3}

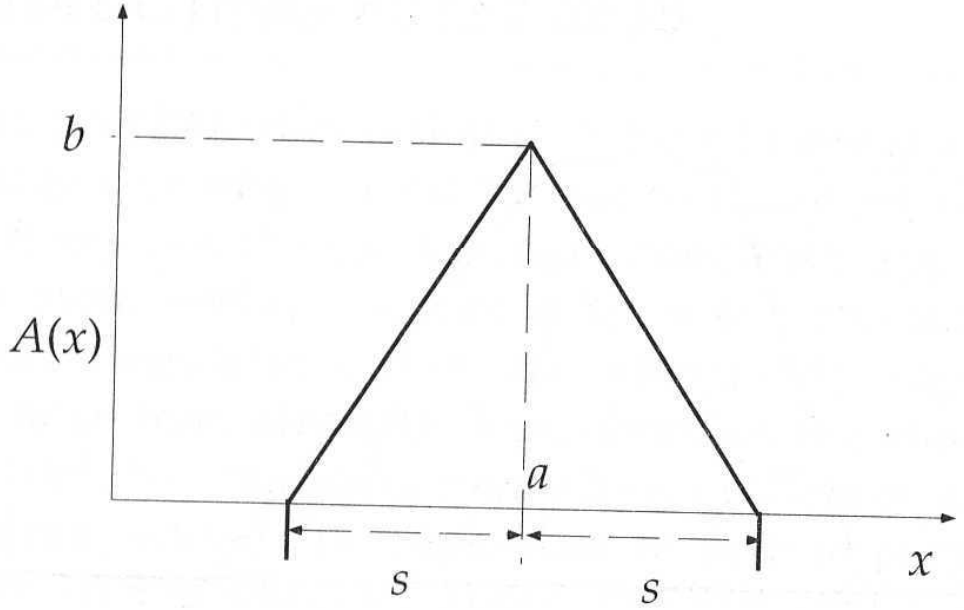
Analytic representation

- Fuzzy set about 6

$$A(x) = \begin{cases} x - 5 & \text{when } 5 \le x \le 6 \\ 7 - x & \text{when } 6 \le x \le 7 \\ 0 & \text{otherwise} \end{cases}$$

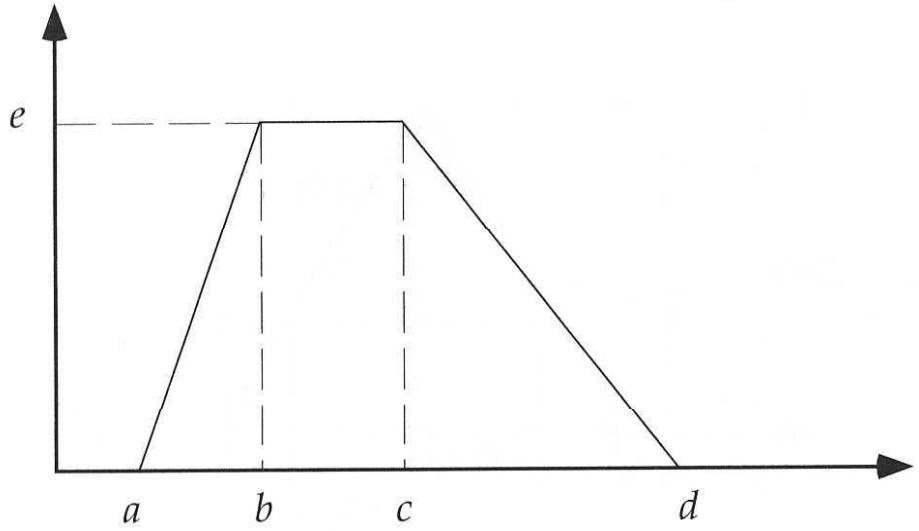


Triangular membership function



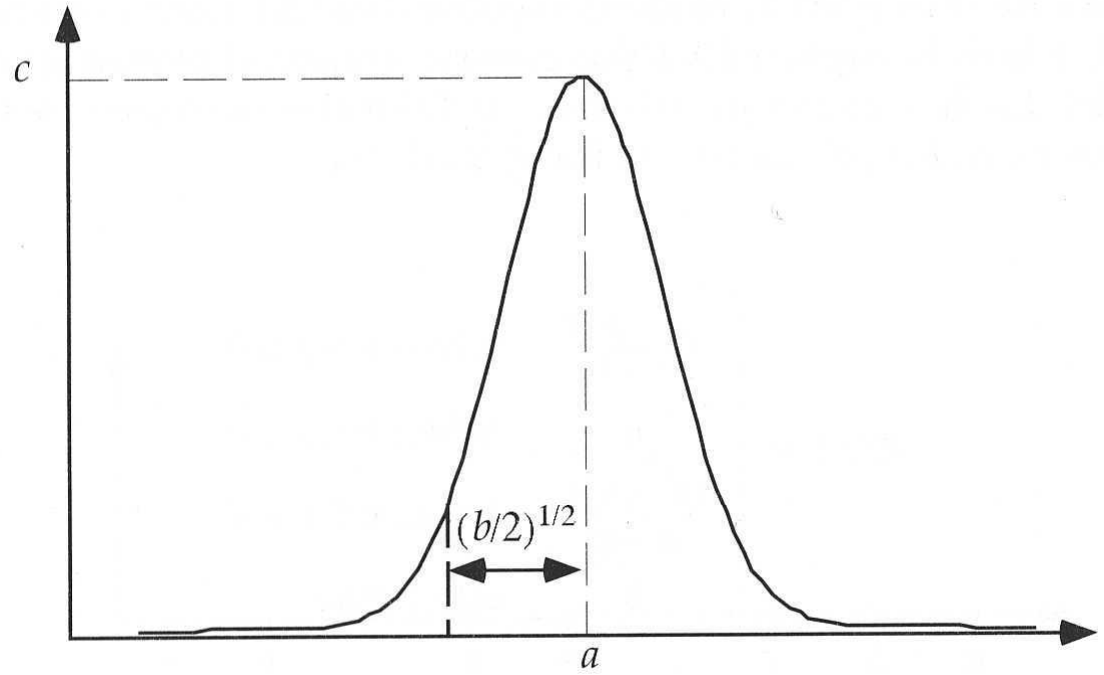
$$A(x) = \begin{cases} b\left(1 - \dfrac{|x-a|}{s}\right) & \text{when } a - s \le x \le a + s \\ 0 & \text{otherwise} \end{cases}$$

Trapezoidal membership function



$$A(x) = \begin{cases} \dfrac{(a-x)e}{a-b} & \text{when a} \le x \le b \\ e & \text{when b} \le x \le c \\ \dfrac{(d-x)e}{d-c} & \text{when c} \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

Bell-shaped membership function



$$A(x) = ce^{-\frac{(x-a)^2}{b}}$$

Constructing fuzzy set

- Construct Set of pairs <x, A(x)>
- Select an appropriate membership function
  - curve-fitting method
  - Neural network

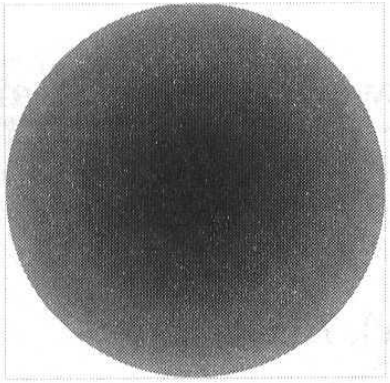
	Alice	Bonnie	Cathy	Dina	Eva
r_1	1	1	1	1	1
r_2	0	0	1	1	1
r_3	0	1	0	1	0
r_4	1	0	1	1	1
r_5	0	0	1	1	1
r_6	0	1	1	1	1
r_7	0	0	0	0	0
r_8	1	1	1	1	1
r_9	0	0	0	1	0
r_{10}	0	0	0	1	0

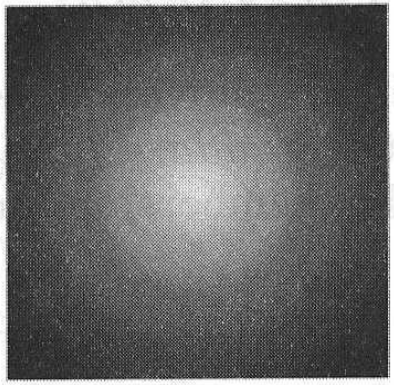
Excellent driver

A =0.3/Alice +0.4/Bonnie +0.6/Cathy +0.9/Dina +0.6/Eva

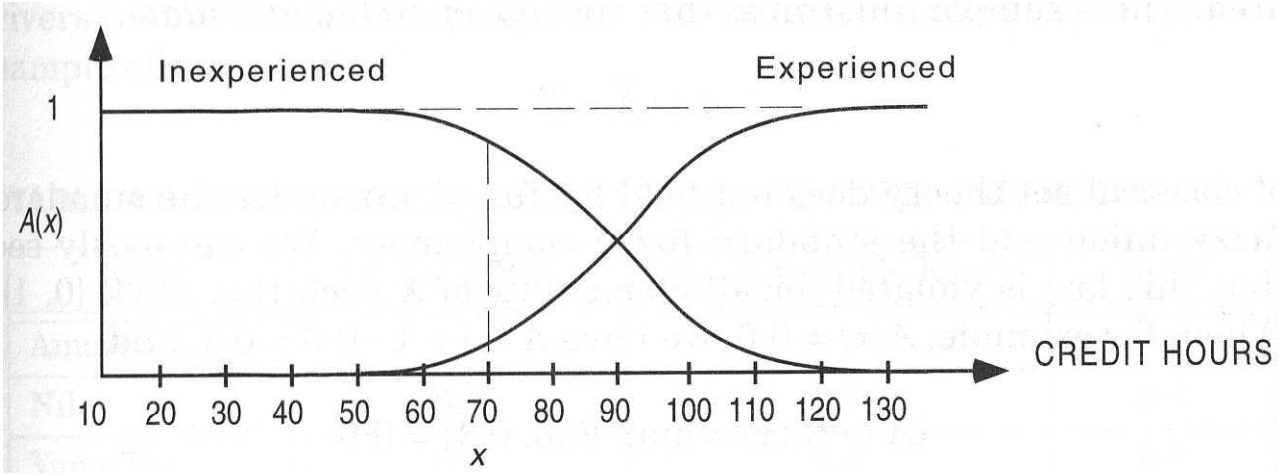
Standard fuzzy complement

- $\tilde{A}(x) = 1 - A(x)$





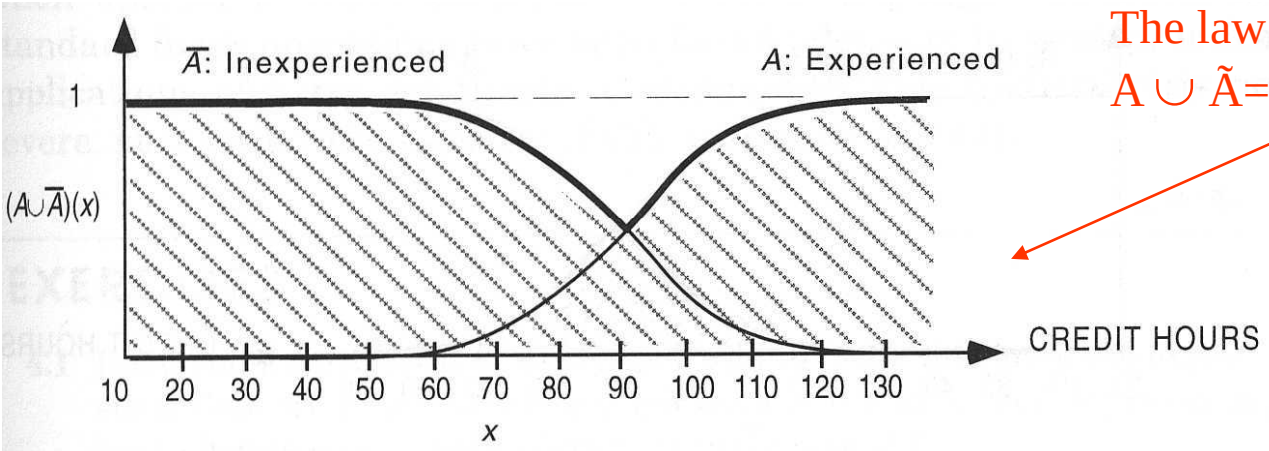
$\overline{A}$



Standard fuzzy union

- $(A \cup B)(x) = \max[A(x), B(x)]$

Patients	*A*=high blood pressure	*B*=high fever	$A \cup B$
1	1.0	1.0	1.0
2	0.5	0.6	0.6
3	1.0	0.1	1.0
...	...	...	...
n	0.1	0.7	0.7

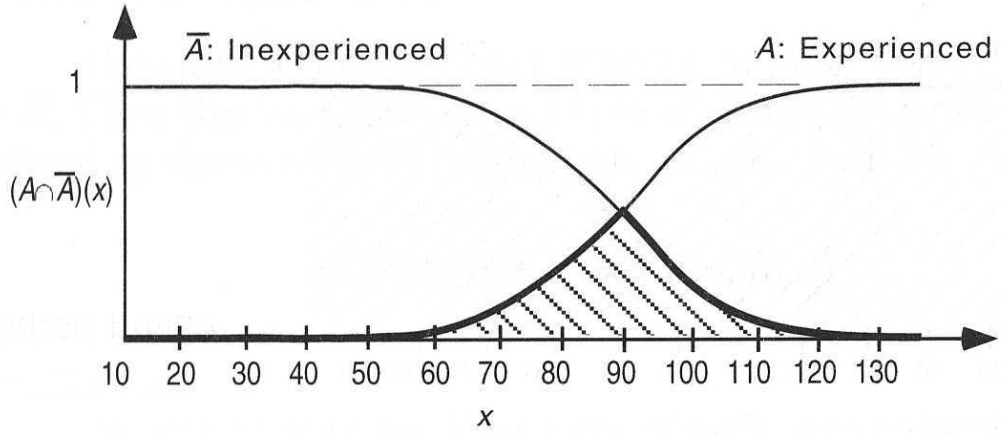


The law of excluded middle,
$A \cup \tilde{A} = X$ does not hold

Standard fuzzy intersection

- $(A \cap B)(x) = \min[A(x), B(x)]$

River	A = Long River	B = Navigable River	$A \cap B$
Amazon	1.0	0.8	0.8
Nile	0.9	0.7	0.7
Yang-Tse	0.8	0.8	0.8
Danube	0.5	0.6	0.5
Rhine	0.4	0.3	0.3



The law of contradiction,
$A \cap \tilde{A} = \varnothing$ does not hold