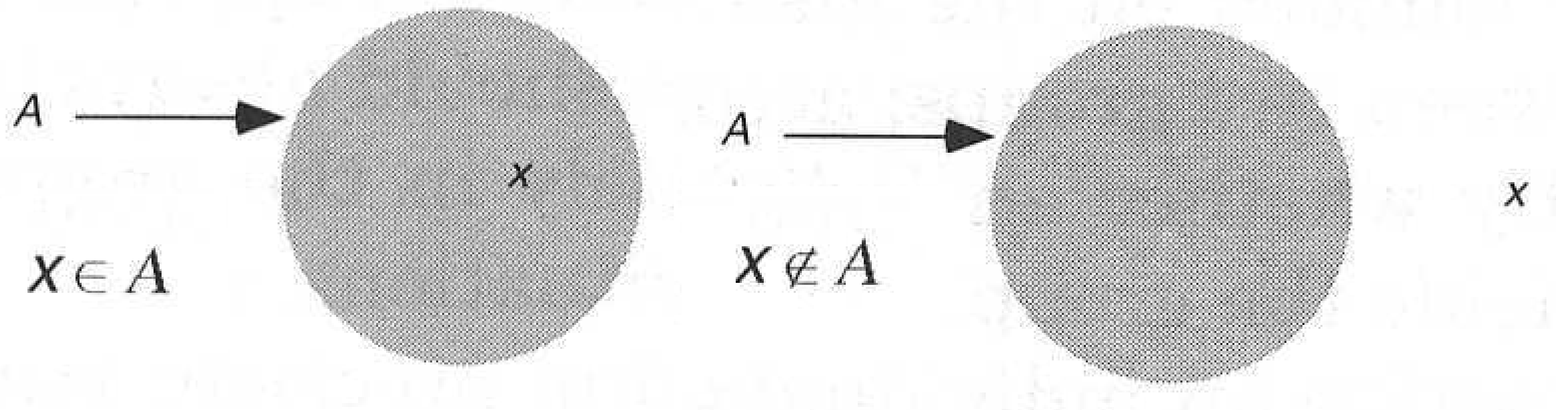
Classical Set Theory



Basic concepts

- Set: a collection of items
- To Represent sets
  - List method A={a, b, c}
  - Rule method $C = \{ x \mid P(x) \}$
  - Family of sets $\{A_i \mid i \in I\}$
  - Universal set X and empty set ∅

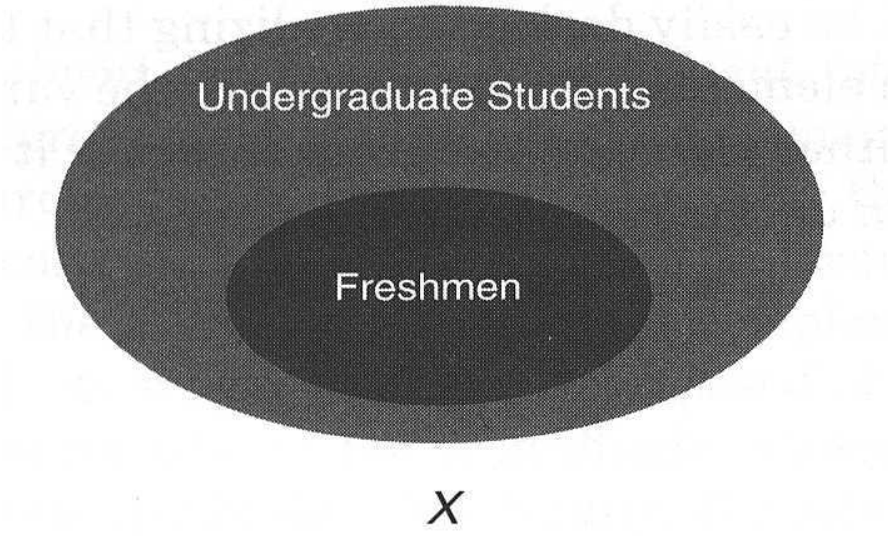
The set C is composed of elements *x*

Every *x* has property P

i: index I: index set

Set Inclusion

- $A \subseteq B$: $x \in A$ implies that $x \in B$ — A is a subset of B
- $A = B$: $A \subseteq B$ and $B \subseteq A$ — A and B are equal set
- $A \subset B$: $A \subseteq B$ and $A \neq B$ — A is a proper subset of B



Power set

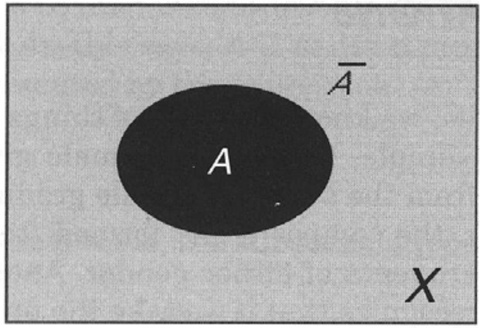
- All the possible subsets of a given set X is call the power set of X, denoted by $\mathcal{P}(X) = \{A | A \subseteq X\}$
- $|\mathcal{P}(X)| = 2^n$ when $|X| = n$

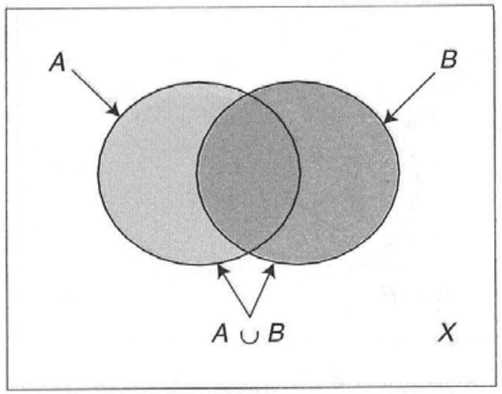
- $X=\{a, b, c\}$

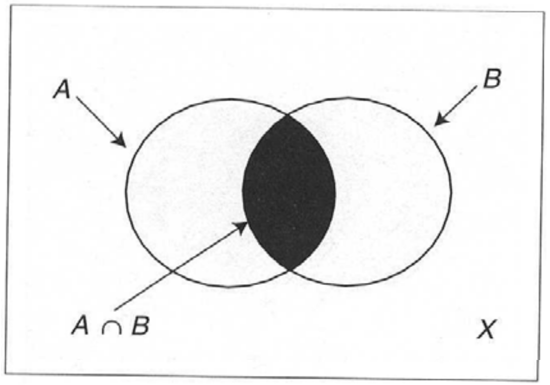
$\mathcal{P}(X) = \{\emptyset, a, b, c, \{a, b\}, \{b, c\}, \{a, c\}, X\}$

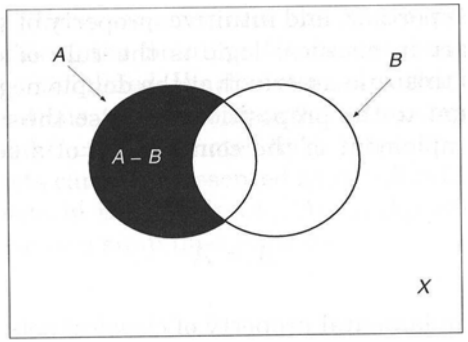
Set Operations

- Complement $\overline{A} = \{x \mid x \in X \text{ and } x \notin A\}$
- Union $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- difference $A - B = \{x \mid x \in A \text{ and } x \notin B\}$







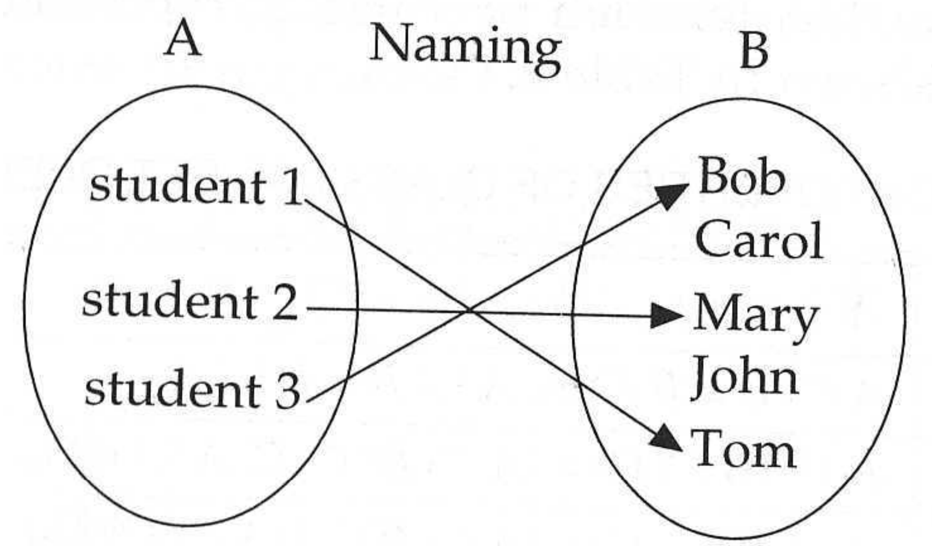


Basic properties of set operations

Involution	$\overline{\overline{A}} = A$
Commutativity	$A \cap B = B \cap A$, $A \cup B = B \cup A$
Associativity	$A \cap (B \cap C) = (A \cap B) \cap C$, $A \cup (B \cup C) = (A \cup B) \cup C$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotence	$A \cap A = A$, $A \cup A = A$
Absorption	$A \cap (A \cup B) = A$, $A \cup (A \cap B) = A$,
Absorption by $\varnothing$ and X	$A \cup X = X$, $A \cap \varnothing = \varnothing$
Identity	$A \cap X = A$, $A \cup \varnothing = A$
Law of contradiction	$A \cap \overline{A} = \varnothing$
Law of excluded middle	$A \cup \overline{A} = X$
De Morgan laws	$\overline{A \cap B} = \overline{A} \cup \overline{B}$, $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Function

- A function from a set A to a set B is denoted by f: A→B
  - Onto
  - Many to one
  - One-to-one

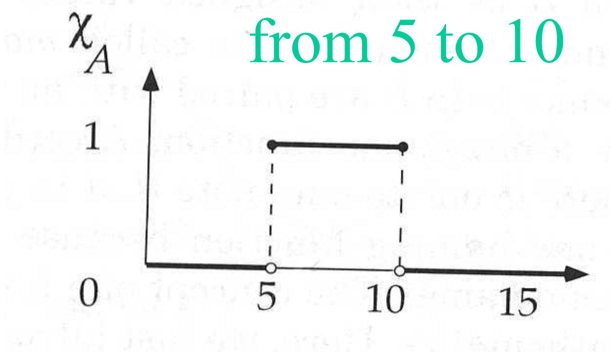


Characteristic function (Belirtev)

- Let A be any subset of X, the characteristic function of A, denoted by χ, is defined by

$$\chi_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

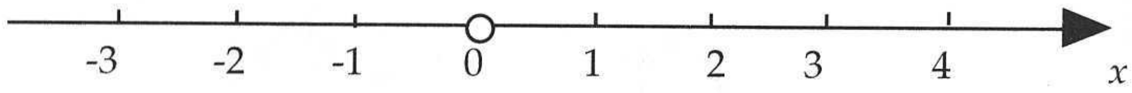
Characteristic function of the set of real numbers from 5 to 10



$$\chi_A(x) = \begin{cases} 1 \text{ if } 5 \leq x \leq 10 \\ 0 \text{ otherwise} \end{cases}$$

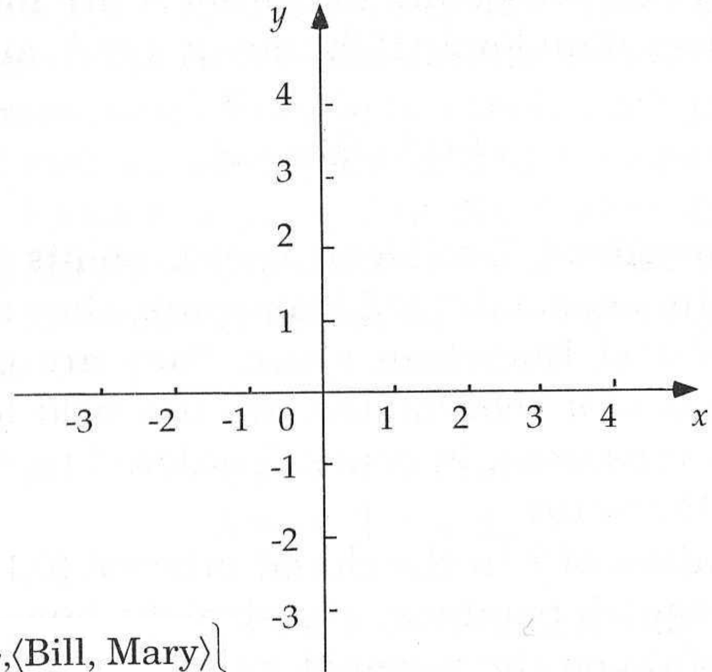
Real numbers

- Total ordering: $a \leq b$
- Real axis: the set of real number $\Re$ (*x*-axis)
- Interval: [a,b], (a,b), (a,b]
- One-dimensional Euclidean space



two-dimensional Euclidean space

- The Cartesian product of two real number
  - $\Re \times \Re$
  - Plane
  - Cartesian coordinate, x-y axes
- Cartesian product
  - A ×B = {<a, b> | a∈A and b ∈B}



$$A = \{\text{John, Bill, Sam}\}$$

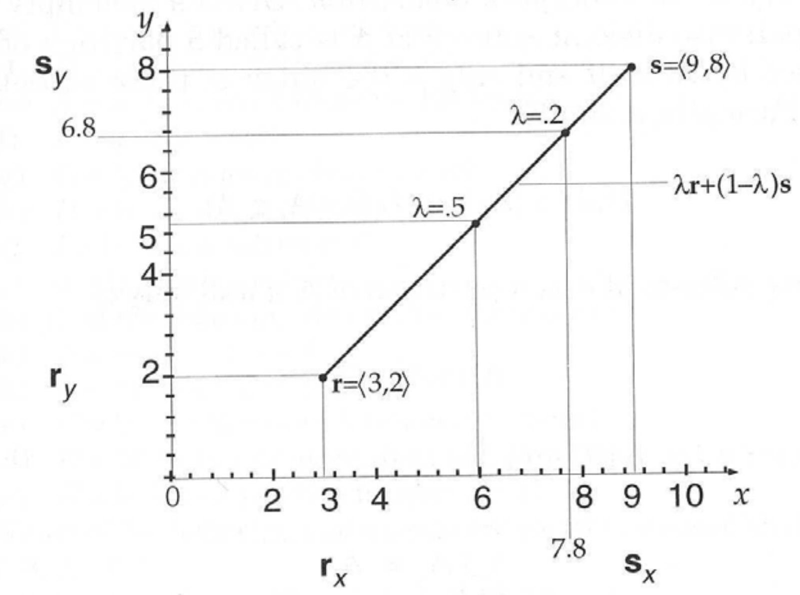
$$B = \{\text{Lisa, Mary, Donna}\}$$

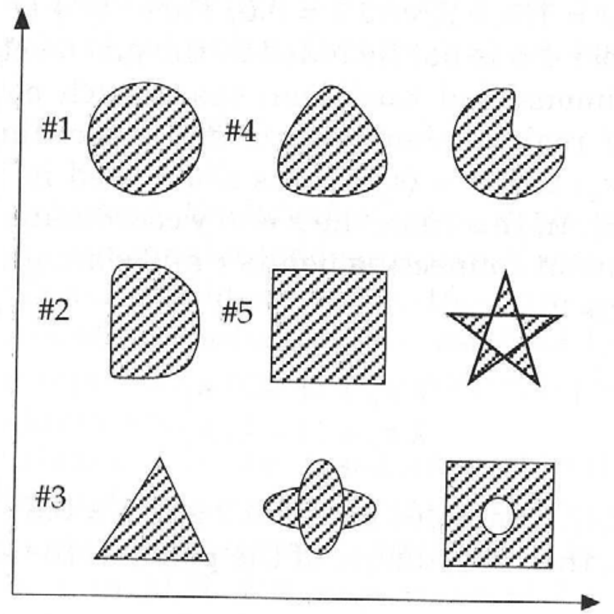
$$A \times B = \left\{ \begin{array}{l} \langle \text{John, Lisa} \rangle, \langle \text{John, Mary} \rangle, \langle \text{John, Donna} \rangle, \langle \text{Bill, Lisa} \rangle, \langle \text{Bill, Mary} \rangle \\ \langle \text{Bill, Donna} \rangle, \langle \text{Sam, Lisa} \rangle, \langle \text{Sam, Mary} \rangle, \langle \text{Sam, Donna} \rangle \end{array} \right\}$$

Convexity (Dışbükey)

- A subset of Euclidean space A is convex, if line segment between all pairs of points in the set A are included in the set.

A is convex $\Leftrightarrow \lambda r+(1-\lambda)s \in A, \forall r,s \in A, \lambda \in [0,1]$





Partition (Bölüntü)

- Given a nonempty set A, a family of pairwise disjoint subsets of A is called a partition of A, denoted by Π(A), iff the union of these subsets yields the set A.
- $\Pi(A) = \{A_i \mid i \in I, \emptyset \neq A_i \subseteq A\} \Leftrightarrow A_i \cap A_j = \emptyset$ for each pair $i \neq j (i,j \in I)$ and $\cup_{i \in I} A_i = A$
- Blocks
- Refinement