Classical (Crisp) Logic

the forms of correct reasoning - formal logic

Symbolic logic

- Definition
  - Language represented by a small set of symbols reflecting the fundamental structure of reasoning with full precision.
- Propositional logic (önermeler mantığı)
- Predicate logic (yüklem mantığı. Nesnelere yüklenen özelliklere ilişkin mantık yordamlar)

$p \wedge q$

$p \Rightarrow q$

Premise(Öncül). Bir çıkarsamanın öncüller kümesini oluşturan önermelerden her biri

Conclusion(Vargı). Varsayımların kanıtlanmasından çıkan sonuç

Forms of reasoning

If today is Monday, then my logic class meets at noon.
Today is Monday.

So, my logic class meets at noon.

If M, then L
M

Therefore, L

□ ⇒ ○
□

∴ ○

Akıl yürütme. İnsan ya da bilgisayar sisteminin belirli varsayımlardan hareketle vargılara ulaşması.

The structure of propositional logic

- Simple proposition
  - A proposition that does not contain any other proposition. (atomic proposition)
  - Önerme. Ya doğru ya da yanlış olan bir sav öne süren bildirim.
- Affirmative (Olumlu) proposition
  - A proposition that contains no negating words or prefixes.

Complex proposition

A dog has four legs and tomorrow is Sunday.

Proposition p

Proposition q

Logic Operations

Operation	Symbol	English Examples
negation	$\neg$	not; it is not the case that, un-; im-; in-
conjunction	$\wedge$	and; but; however
disjunction	$\vee$	or; unless
implication	$\Rightarrow$	if, … then; implies; only if
equivalence	$\Leftrightarrow$	if and only if; when and only when

Negation (Olumsuzlama)

- p = 『a dog has four legs』
- q = 『Elvis is mortal』

English Sentence	Symbolic Representation
A dog does NOT have four legs	$\neg p$
Elvis is immortal	$\neg q$

p	$\neg p$
F	T
T	F

← Truth table →

p	$\neg p$
0	1
1	0

fuzzy database modeling

Conjunction (Birletim; Mantıksal çarpım, VE işlemi)

Today is Wednesday and tomorrow is Thursday.

Binghamton is a town and New York is a state.

John graduated summa cum laude; moreover, he received a poetry prize.

$$|p \wedge q| = \min [|p|, |q|]$$

$$|p \wedge q| = |p| \cdot |q|$$

$$|p \wedge q| = \max [0, |p| + |q| - 1]$$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Disjunction (Ayırtım, Mantıksal toplam, VEYA işlemi)

Jupiter has ten moons or it is a massive planet.
A double major or a high grade point average is impressive for law school.
John plays football or basketball.

$$|p \vee q| = \max\,[|p|,\ |q|]$$

$$|p \vee q| = \min\,[1, |p| + |q|]$$

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Implication (Gerektirme, içerme)

If I meet the university distribution requirements, then I can graduate.
On condition that the vehicle is maintained properly, it will run for at least 200,000 miles.
We will have a celebration only if we are victorious.

antecedent — consequent

$$|p \Rightarrow q| = \min(1, 1 + |q| - |p|)$$
$$|p \Rightarrow q| = 1 - |p|(1 - |q|)$$

$$|p \Rightarrow q| = |\neg p| \vee |q|$$

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

İki önerme verildiğinde ancak birinci önerme doğru, ikinci önerme yanlış olduğunda çıktının doğru olduğu mantık işleci.

Equivalence (Eşdeğerlik, denklik)

You will become a star if and only if you have an enterprising agent.

if p, then q, and if q, then p

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$

$$|p \Leftrightarrow q| = |p| \cdot |q| + |\neg p| \cdot |\neg q|$$

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

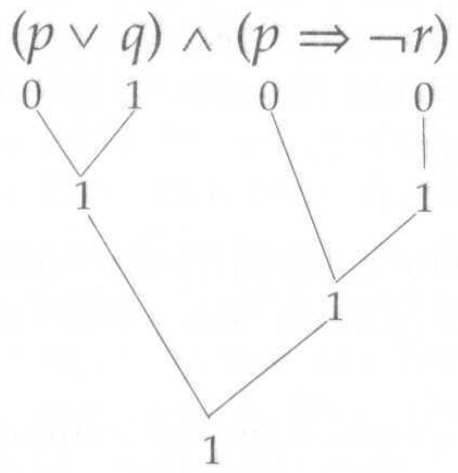
İki p ve q önermesi verildiğinde, her ikisinin de doğru (mantıksal 1) ya da her ikisinin de yanlış (mantıksal 0) olduğunda, çıktısı doğru (1) olan ikili Boole işleci.

Truth values of complex propositions (önerme)

Either Joe or Bill or both will play badly at the tournament, but if Joe plays badly, then there will be no splashy victory party.

$$(p \vee q) \wedge (p \Rightarrow \neg r)$$

p: Joe plays badly at the tournament.
q: Bill plays badly at the tournament.
r: There will be a splashy victory party.



Evaluation of a logic expression.

Table of a complex proposition

p	q	r	$(p \vee q) \wedge (p \Rightarrow \neg r)$
0	0	0	0 **0** 1 1
0	0	1	0 **0** 1 0
0	1	0	1 **1** 1 1
0	1	1	1 **1** 1 0
1	0	0	1 **1** 1 1
1	0	1	1 **0** 0 0
1	1	0	1 **1** 1 1
1	1	1	1 **0** 0 0
			c d b a

Contradictions and Tautologies (Çelişki, tutarsızlık ve Gereksiz tekrar)

Contradictions

p	$p \wedge \neg p$
0	0
1	0

Tautologies

p	$p \vee \neg p$
0	1
1	1

Tautology:logical implication

p	q	$[(p \Rightarrow q)$	$\wedge$	$p] \Rightarrow q$
0	0	1	0	**1**
0	1	1	0	**1**
1	0	0	0	**1**
1	1	1	1	**1**
		a	b	c

Tautology:logical equivalence

p	q	$\neg(p$	$\wedge$ $q)$	$\Leftrightarrow$	$(\neg p$	$\vee$	$\neg q)$
0	0	1	0	**1**	1	1	1
0	1	1	0	**1**	1	1	0
1	0	1	0	**1**	0	1	1
1	1	0	1	**1**	0	0	0
		f	e	g	b	c	a

Logic functions

Function	Name	Expression Examples
r_1	tautology	$[(p \Rightarrow q) \wedge p] \Rightarrow q,$
r_2	disjunction	$p \vee q,\ \neg(\neg p \wedge \neg q)$
r_3	implication	$q \Rightarrow p,\ \neg q \vee p$
r_4	assertion	$p \vee (p \wedge q)$
r_5	implication	$p \Rightarrow q,\ \neg p \vee q$
r_6	assertion	$q \vee (p \wedge q)$
r_7	equivalence	$p \Leftrightarrow q,\ (p \wedge q) \vee (\neg p \wedge \neg q)$
r_8	conjunction	$p \wedge q$
r_9	not both	$\neg (p \wedge q)$
r_{10}	nonequivalence	$(p \vee q) \wedge \neg (p \wedge q)$
r_{11}	negation	$\neg [q \vee (p \wedge q)],\ \neg q$
r_{12}	inhibition	$p \wedge \neg q, \neg (p \Rightarrow q)$
r_{13}	negation	$\neg [p \vee (p \wedge q)],\ \neg p$
r_{14}	inhibition	$q \wedge \neg p,\ \neg (q \Rightarrow p)$
r_{15}	neither–nor	$\neg (p \vee q)$
r_{16}	contradiction	$(p \wedge \neg p) \wedge (q \wedge \neg q)$

Valid inference

Premise 1	$(p \Rightarrow q) \wedge (r \Rightarrow s)$
Premise 2	$p \vee r$
Conclusion	$q \vee s$

p	q	r	s	$(p \Rightarrow q) \wedge (r \Rightarrow s)$	$p \vee r$	$q \vee s$
0	0	0	0	**1**	**0**	**0**
0	0	0	1	1	0	1
0	0	1	0	**0**	**1**	**0**
0	0	1	1	1	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	0	1	1
0	1	1	1	1	1	1
1	0	0	0	**0**	**1**	**0**
1	0	0	1	0	1	1
1	0	1	0	**0**	**1**	**0**
1	0	1	1	0	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	0	1	1
1	1	1	1	1	1	1

Invalid inference

error

Premise 1	$p \Rightarrow q$
Premise 2	$\neg p$
Conclusion	$\neg q$

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$
0	0	1	1	1
0	1	**1**	**1**	**0**
1	0	0	0	1
1	1	1	0	0

Basic Inference forms

Conjunction (Conj.) 1. p 2. q $\therefore p \wedge q$	*Simplification (Simp.)* 1. $p \wedge q$ $\therefore p$
Addition (Add.) 1. p $\therefore p \vee q$	*Disjunctive Syllogism (DS)* 1. $p \vee q$ 2. $\neg p$ $\therefore q$
Modus Ponens (MP) 1. $p \Rightarrow q$ 2. p $\therefore q$	*Modus Tollens (MT)* 1. $p \Rightarrow q$ 2. $\neg q$ $\therefore \neg p$
Constructive Dilemma (CD) 1. $(p \Rightarrow q) \wedge (r \Rightarrow s)$ 2. $p \vee r$ $\therefore q \vee s$	*Destructive Dilemma (DD)* 1. $(p \Rightarrow q) \wedge (r \Rightarrow s)$ 2. $\neg q \vee \neg s$ $\therefore \neg p \vee \neg r$
Hypothetical Syllogism (HS) 1. $p \Rightarrow q$ 2. $q \Rightarrow r$ $\therefore p \Rightarrow r$	*Absorption (Abs.)* 1. $p \Rightarrow q$ $\therefore p \Rightarrow (p \wedge q)$

Rules of Replacement

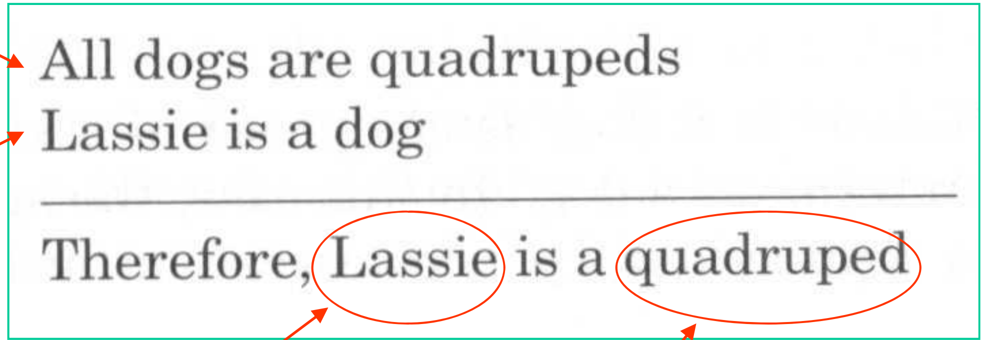
Involution (Double Negation)	$p \Leftrightarrow \neg\neg p$
Commutativity	$(p \wedge q) \Leftrightarrow (q \wedge p)$ $(p \vee q) \Leftrightarrow (q \vee p)$
Associativity	$[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$ $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$
De Morgan's Laws	$\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
Distributivity	$[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$ $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
Equivalence	$(p \Leftrightarrow q) \Leftrightarrow [(p \Rightarrow q) \wedge (q \Rightarrow p)]$ $(p \Leftrightarrow q) \Leftrightarrow [(p \wedge q) \vee (\neg p \wedge \neg q)]$
Contraposition	$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$
Implication	$(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$
Exportation	$[p \Rightarrow (q \Rightarrow r)] \Leftrightarrow [(p \wedge q) \Rightarrow r]$
Idempotency	$(p \wedge p) \Leftrightarrow p$ $(p \vee p) \Leftrightarrow p$

Predicate Logic

yüklem mantığı. Nesnelere yüklenen özelliklere ilişkin mantık yordamları.

General Proposition

Singular Proposition


All dogs are quadrupeds
Lassie is a dog

Therefore, Lassie is a quadruped

Subject term

Predicate term

Singular Propositions

Lassie is a dog

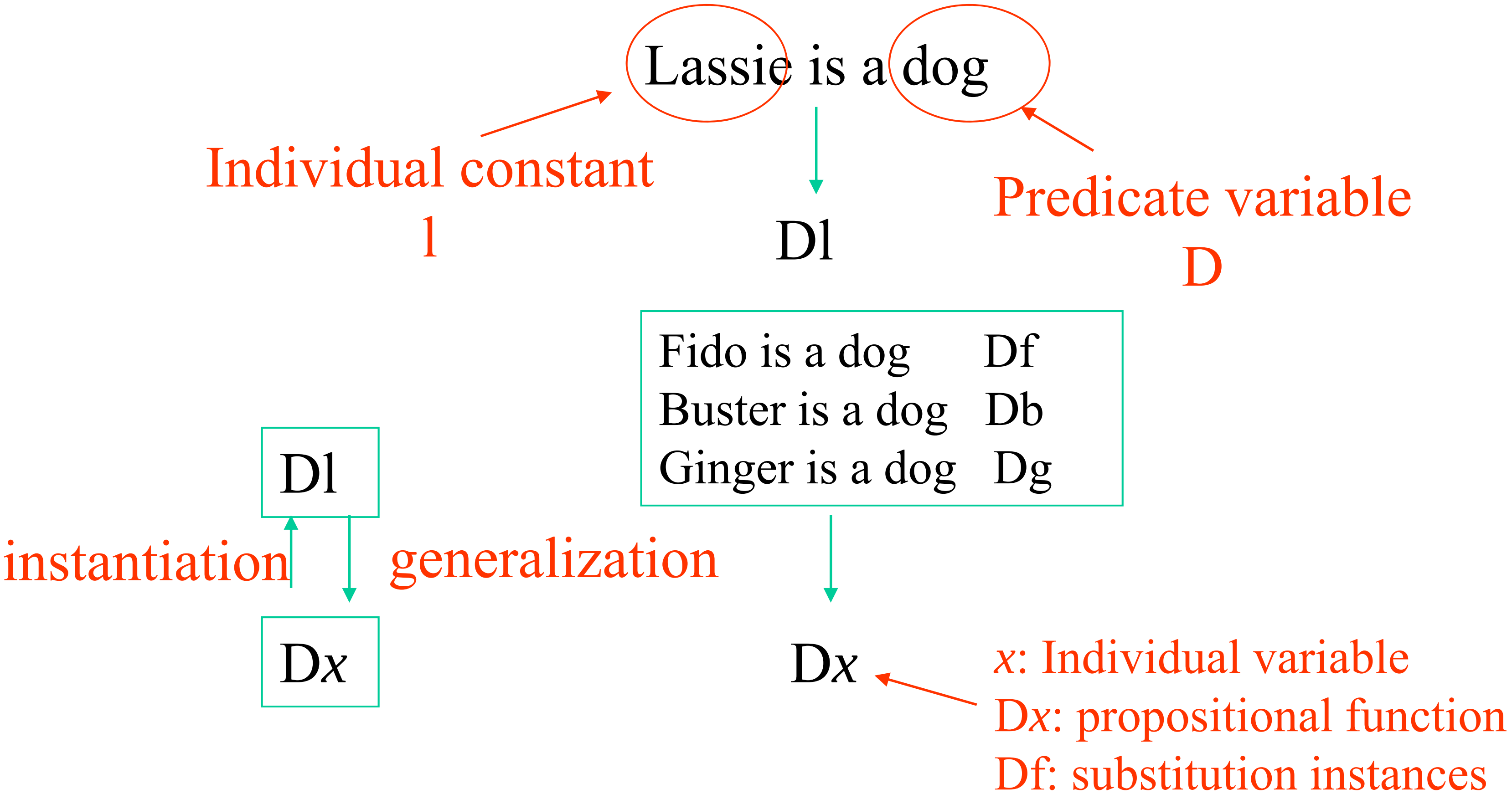
Individual constant
l

Dl

Predicate variable
D

Fido is a dog	Df
Buster is a dog	Db
Ginger is a dog	Dg

Generalization



General Propositions

$(\exists x)Dx$: There exists at least one x, such that the x is a dog
Existential generalization $\exists x$: Existential quantifier

$(\exists x)(Dx \wedge Qx)$: There exists at least one thing, such that it is both a dog and a quadruped.

$(\forall x)\ Dx$: For any x, x is a dog
universal generalization $\forall x$: universal quantifier

$(\forall x)\ Dx \Rightarrow Qx$: for any x, if x is a dog, then x is a quadruped

All dogs are quadrupeds
Lassie is a dog

Therefore, Lassie is a quadruped

Relations represented by predicate logic

- John loves Mary --- Ljm

L ：relation j,m ：individual constant

- Everything is attracted by something --- $(\forall x)(\exists y)Ayx$

x y

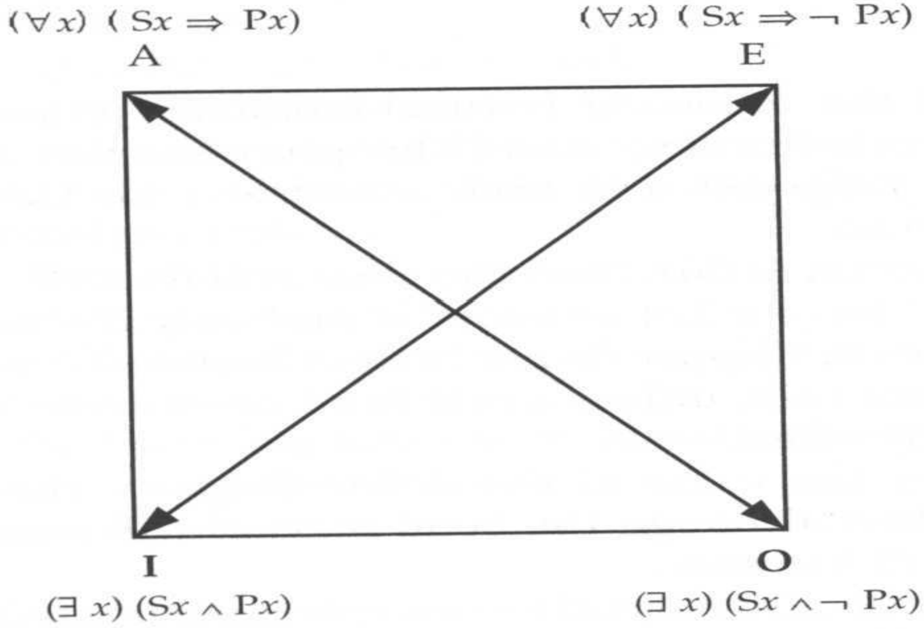
Quantifier Negation

- It is false that everything is square --- $\neg(\forall x)Sx$
- There is something which is not square --- $(\exists x)\neg Sx$

Quantifier negation equivalences

$(\forall x)\ Px$	$\Leftrightarrow$	$\neg(\exists x)\neg Px$
$\neg(\forall x)\ Px$	$\Leftrightarrow$	$(\exists x)\neg Px$
$(\forall x)\neg Px$	$\Leftrightarrow$	$\neg(\exists x)\ Px$
$\neg(\forall x)\neg Px$	$\Leftrightarrow$	$(\exists x)\ Px$

The Square of Opposition



Universal affirmative (A)	"all S are P"	$(\forall x)\ (Sx \Rightarrow Px)$
Universal negative (E)	"no S are P"	$(\forall x)\ (Sx \Rightarrow \neg Px)$
Existential affirmative (I)	"there exists at least one S which is P"	$(\exists x)\ (Sx \wedge Px)$
Existential negative (O)	"there exists at least one S which is not P."	$(\exists x)\ (Sx \wedge \neg Px)$